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CONTINUOUS,

REVOLVING DRAWBRIDGES:

THE PRINCIPLES OF THEIR CONSTRUCTION AND THE CALCULATION OF THE STRAINS IN THEM.

WITH MORE ESPECIAL REFERENCE TO THE DESIGNING OF
CONTINUOUS *PANEL* GIRDERS OF THIS DESCRIPTION.

By CLEMENS HERSCHEL,
CIVIL ENGINEER, MEMBER OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS.

WITH NINETEEN WOODCUTS AND TEN HELIOTYPE PLATES.

BOSTON:
LITTLE, BROWN, AND COMPANY.
1875.

Cambridge:
Press of John Wilson & Son.

TO

JOS. P. DAVIS, Esq.,

CITY ENGINEER OF BOSTON, MASS.,

AS THE INNOCENT CAUSE OF ITS HAVING BEEN WRITTEN,

THIS CONTRIBUTION TO THE ADVANCEMENT OF BRIDGE BUILDING

IS RESPECTFULLY DEDICATED,

WITH GREAT APPRECIATION OF HIS TALENTS AS A CIVIL ENGINEER,

BY

THE AUTHOR.

PREFACE.

THIS little book, in its original shape, was a paper written for the AMERICAN SOCIETY OF CIVIL ENGINEERS, and was prompted by the author's design of the Quinnipiac Bridge, at New Haven, Conn., and by his employment in testing a design for the Eastern Avenue bridge, in Boston, Mass. It has been thought that in its present shape it may be of some use to students in engineering construction; to experts also, on the subject of continuous girders, at home and abroad, so much of the work as relates to subjects, that, it is believed, are here for the first time treated, it may not be without interest. In this connection, the author will refer only to his method of first calculating continuous girders for given reactions, and then weighing off these same reactions to act under the girders in setting them up on their supports; to those parts of the book that treat of the extension of this idea to the calculation of continuous girders, by first dividing up the total load into parts that make certain reactions $=0$, and those that act on the girder as though it were continuous over all its supports; and to the cases in which two supports are themselves carried and pivoted upon a third support.

CLEMENS HERSCHEL.

BOSTON, January 27, 1875.

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CONTINUOUS,

REVOLVING DRAWBRIDGES.

First principles, then rules.

In order that the human intellect may approach the perfect answer in any physical investigation, there is necessary the combined application, each in its proper proportion, of three instruments: mathematics, experiment, and a well-balanced, trained, common-sense judgment.

INTRODUCTORY. — The choice of subject above presented probably needs no apology, before an audience of American engineers. After having neglected the construction of *continuous* fixed spans, with probably good reason in the majority of cases, as we shall see, but on the other hand carried the construction of certain kinds of single fixed spans (link-bridges) to a degree of perfection and size hitherto unknown, American engineers have suddenly been brought face to face with the problem of continuous spans, by the abandonment of the old form of drawbridge, whose ends are suspended from a central tower by chains or rods, and the adoption of the continuous form of truss for such structures. The number of drawbridges of this latter class is steadily increasing, and some of them have already attained a magnitude of span never before attempted; yet it is a painful fact that, in the calculation of the strains in continuous drawbridges, unsupported opinion and "good enough" approximations have often had a controlling voice; whereas such guides are evidently out of place in engineering or in any other physical science. And it is, perhaps, only a natural consequence of this that there are many unpleasant rumors (facts are difficult to arrive at in such cases) as to the unsatisfactory performances, wear, and life of many of the great drawbridges of the country, both of the old and new styles.

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* The word "drawbridge" will, in this paper, be used only in the sense of one of that species of drawbridges that are made to revolve around a vertical axis situated between their ends.

The subject is therefore submitted as one of practical and ample pecuniary interest to the profession and to their employers.

PART I.

§ 1. HISTORY OF THE THEORY OF CONTINUOUS GIRDERS. — The calculation of continuous girders commences with Navier, about 1830, who, it is believed, first propounded the theory of the "elastic line;" that is, the curve or form which the neutral axis of a body, following the laws of elasticity, would assume upon being supported by any number of fixed supports, which were first taken as being all on the same level, and then acted upon by a series of vertical forces or loads. Navier showed how, from the equation of this curve, could be deduced the reactions at each support, the value of the moments over the supports and generally along the line, &c. Still, his method led to somewhat laborious calculations, and was therefore but little used, until in December, 1857, a French engineer, Clapeyron, published, in the "Comptes Rendus," an improvement on the method of Navier, which consisted in finding primarily the moments over each support, and then deducing, from the moments so found, the reactions sought for. Clapeyron's equations required that: 1. The load be uniformly distributed on each span. (The several spans may, however, have different loads.) 2. The supports must all be situated on one and the same level. 3. The moment of inertia of the cross-section of the girder, referred to the neutral axis, must be constant. Other assumptions made by Clapeyron, but which are common to all the theories of the elastic line, before and since his day, will be spoken of further on.

The publication of Clapeyron's article seems to have given a great impetus, in France and Germany at least, to the investigation of the properties of continuous girders, and speedily led to the extension of the theory of the elastic line to the cases where the supports are not on a level,* to investigations as to the effect produced by such a change of level in the supports, &c.; and much has since been written in French, German, and English * technical journals, and other publications, with some books partly or wholly on this subject. [See the list given in the Appendix.]

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* First noticed by C. Köpke in "Zeitschrift des Archt. & Ing. Vereins zu Hannover," 1856.

It is characteristic, however, of all the books and articles that appeared on this subject, between 1860 and 1873, that they severally treat of special cases only, at least what in the light of the most recent work † appear as only special cases. Thus Mohr develops the properties of girders, continuous over two or three spans of any relative length, for *uniformly distributed* loads, with the supports on a level, or raised or lowered certain known quantities, and for girders of varying section. Laissle and Schuebler do nearly the same; while Quensel gives the equations for *concentrated* loads, in girders continuous over two spans, when both spans are equal.

It was the first intention of the writer to add to these cases, by developing the equations for concentrated loads, on a girder, continuous over two spans, when the spans are *unequal* and the supports either in or out of level; which then seemed the only set of equations needed to meet the general case here treated of, — a continuous girder drawbridge, loaded and supported in any manner.‡ All such partial investigations have been set at rest, however, by the recent work of Weyrauch, previously referred to, which, in the language of the preface, presents a complete "theory of straight girders, continuous over any number of openings from 1 to ∞, and for any kind of concentrated or regularly or irregularly distributed loads." It even treats the single span girder as but a special case of the general subject above presented. Following a work as exhaustive as this, there remains but little to be done in the way of generalization; and it has been freely used in deducing the equations needed to calculate the reactions at the several points of support of a drawbridge.

The general equations thus taken from Dr. Weyrauch's book, the special equations then derived from them, and their mode of derivation, will be found in Part II.

§ 2. THE PRACTICE AND THEORY OF THE ELASTIC LINE. — It is time now to speak of the value of all these investigations, of the assumptions upon which they rest, and in how far we can judge or *know* that the results obtained represent the exact truth.

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* The writer cannot state as to works in other than these three languages.

† The work by Dr. Jacob I. Weyrauch cited in the Appendix.

‡ However, this would not have met the whole case, as will presently appear; for usually the drawbridge, owing to its being supported at *two* points at the centre, becomes a girder continuous over four supports, or of three spans instead of two.

The theory of the "elastic line," together with the long train of investigations that has grown from it, rest upon certain assumptions to be found in most theories of flexure, and very fully set forth in the work to which reference has just been made. In order that a bridge structure should exactly conform to these assumptions: 1. Every member and every part of every member would have to extend or compress strictly equal distances by the application of equal strains per unit of section. 2. If the girder were laid over on its side, — that is, not be acted on by what ordinarily are its vertical forces, — every part of it would have to be without any strain whatever. 3. When set up, no part of it must in the slightest yield otherwise than according to 1. 4. Should it act as a continuous girder on supports that are on a level, its actual supports must, when the bridge is set up, conform strictly to the profile of the bottom of the bridge when it is in condition 2. This is a point of some importance, as will appear; and it is well perhaps at once to clearly catch the idea of what is meant by "supports on a level" and "supports so and so much out of level," speaking after the manner of continuous girders. In point of *absolute* level, the supports may all be on a different grade and yet react as supports that are on a level, if the girder, when without strain or when it is acted upon by the calculated theoretical reactions, has an under profile fitting to this broken grade line of the supports; and, if mention should be made of such a support being lowered so much out of level, it would mean, lowered so much from this primary grade line, &c. 5. In the theories generally given it is assumed that the moment of inertia of the cross section of the girder referred to the neutral axis is a constant. There are, to be sure, methods for calculating the reactions, moments, and strains in girders whose cross sections, and therefore moments of inertia, vary by steps, or according to known gradual increments or decrements, accurately as well as approximately, but they are both so laborious as to make their use unwarranted, save in rare cases; and even then the uncertainty and approximations inherent to the theory in general are not removed by this additional accuracy of taking into account these variations. It is safe to affirm, namely, that in actual practice none of the conditions 1, 2, 3, and 4 ever strictly obtain. Under 1, different pieces of iron do not extend and compress strictly as the strains upon them; each separate plate, angle, T, or channel will vary more or less in this respect within its own length and each with the other. 2 is a condition which must be different in different forms of construction, link or riveted, or nut and rod, and is probably never absolutely attained in practice. 3 depends on 2 in the first place, and even starting with 2 perfectly attained it is just as reasonable to suppose that more or less members will yield somewhat, no matter how little, other than by extension or compression of the material they are composed of, as that they will not at all, especially on being first brought into action; and, again, that different forms of construction will act differently in this respect. 4 is a condition of which, ordinarily, really nothing is absolutely known; it is of the more importance, as very small changes of level produce a great increase or diminution of strains. How this condition may be regulated will be shown farther on. 5 has already been discussed.

There would seem to be, therefore, no particular reason why we should give implicit faith to the results of the theories based on that of the elastic line, any more than we do to those derived from theoretical hydraulics; we can, on the contrary, as a matter of judgment, gravely question their accuracy, and, so far as known to the writer, there are yet wanting any experiments on finished structures from which coefficients of correction, such as are indispensable in the science of hydraulics, could be deduced to apply to the science of bridge-building, more particularly to that of continuous girders.

Another difficulty lies in the form of the equations presented to the engineer for calculating the dimensions of a desired girder. If the supports are not "on a level," or if it is desired to apply the more exact formulæ, which make allowance for the variations in the cross section of the chords, it will be found that these very equations already contain the desired dimensions, algebraically of course. There is then nothing left to do but to apply the method of successive approximations; that is, first to find the dimensions as well as may be, introducing the value of the dimensions sought in the equation used to find them, according to the best judgment of the calculator, then from the structure so designed to start anew. There is no desire here to overrate the deficiencies of the system as it stands developed to-day, nor to conceal the fact that multitudes of bridges have been built in accordance therewith and are doing good service. The question that remains is: inasmuch as the formulæ do not give exact mathematical results, how near are they to the truth for various kinds of bridges or what are the coefficients of correction? Happily, much uncertainty could be eliminated, and the two last described calculations dispensed with, were the method of *weighing off the actual reactions* of the finished bridge once substituted for that of calculating the proper position or level of the supports, in order that certain reactions might be obtained. To arrive at this method in the development of the proper manner of calculating drawbridges seems very natural, but it is nevertheless somewhat puzzling that, with the great care and expense applied to the erection of the numerous continuous girders that have been built in France and Germany, this simple method should not have been thought of and practised rather than the uncertain one of calculating minute differences of level of the supports, and then attaining them as nearly as possible.

To illustrate, take a girder of uniform section, uniformly loaded, of two equal spans, supported on three supports all on a level.

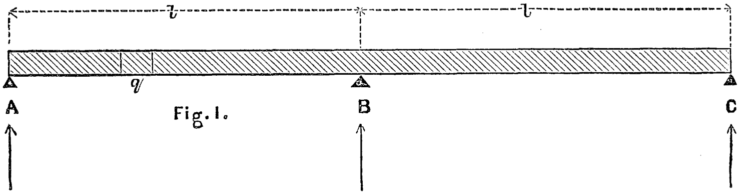

Fig. I.

A, B, C, are the reactions, q the weight per unit of length, therefore from **49** and **50**,* $A = C = \frac{3}{8}\,ql$, $B = \frac{10}{8}\,ql$, when all the conditions of a continuous girder are fulfilled. But if A and C are each lifted up by a lever or set of levers (the levers ultimately to be kept in position by, say a spring dynamometer) or by any other weighing apparatus more or less approximate,† until these scale beams read $\frac{3}{8}\,ql$, and permanent wedges are then driven under A and C until the spring dynamometers or temporary supports return to indicate 0, we shall be certain that we have left the girder acting as a theoretically perfect continuous girder, under that load at least. Again, if the reactions left under the girder were only those for which it had been calculated, it would be a correctly designed structure, no matter whether these reactions were those theoretically due to a continuous girder on level supports or on supports to a certain degree out of level. For a load q_1 on one span, and q_2 on the other, the theoretical reactions on level supports would be (**46**—**48**)—

$$A = \frac{l}{16}\,(7q_1 - q_2),\ B = \frac{10\,l}{16}\,(q_1 + q_2),\ C = \frac{l}{16}\,(7q_2 - q_1),$$

and further experiment would show whether the same girder, reacting theoretically perfect under the one kind of load, did so under the other kind also, or, if not, what were the actual reactions. From a series of such tests, valuable practical data would speedily be obtained; and in the mean while girders would have to be so supported that they should receive the proper reactions when under the loads which produce the most frequently occurring maximum strains.

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* See equations in Part II.

† A good hydraulic press that had previously been experimented with and duly rated would make a convenient and direct weighing machine.

§ 3. SOME PROPERTIES OF CONTINUOUS GIRDERS. — Before proceeding to speak specifically of only continuous drawbridges, it may be well to become more fully acquainted with some of the properties of continuous girders in general. One of their most remarkable characteristics is the effect they undergo by being supported on supports which are not "on a level." The equations that give the reactions, maximum moments, &c., under these conditions, all contain the moment of inertia of the girder cross section and the ordinates below a level line of the several points of support. Likewise, with given sections of girder, spans, and loads, may be found the ordinates of supports that will produce desired strains and moments. See equations 32–34 containing the term Y, or 59–62 containing Y_1 and Y_2, and others. A large part of most treatises is taken up with the calculation of these special ordinates; that is, ordinates producing special strains and moments, and the result of some of these calculations it will be instructive to look over. It is proposed in this paper to make no use of any formulæ containing the terms Y — that is, the ordinates of the support and the moment of inertia of the girder cross section — for several reasons: 1. to abbreviate calculation; 2. because these ordinates cannot be measured practically with the nicety that the equations demand, — thousandths of a foot in difference of level usually make tons of difference of reaction and many tons' difference of strain; 3. because the introduction of these terms, as has been stated, fits the equations indeed for finding the strains in a designed girder, but not for finding the correct dimensions of a girder about to be designed; 4. because by directing our energies toward a different object — the weighing of reactions — more can be achieved in the investigation of the same subject. In the present state of the science of continuous girders, certainly the value of the deductions about to be given lies rather in the warning they offer as to the effects of accidental or unknown differences of level of the supports, than in the use to be made of them and the formulæ they are derived from, in the designing of new works. The following tables, I. for two spans and II. for three spans, are taken from the work of Laissle and Schuebler, and will explain themselves. The object aimed at is to make the cross section of the chords as nearly uniform as possible; that is, equal in maximum in each span to what it is over the centre support in the case of two spans, and equal in all three spans to what it is over the centre supports in the case of three spans.

TABLE I. (*for* 2 *Spans*).

	Length in ft. of each span	23.	32.8	49.2	65.6	98.4	131.2	196.8	328.
	Ratio $\frac{p}{q}$.	$\frac{1}{9}$	$\frac{1}{7}$	$\frac{1}{5.3}$	$\frac{1}{4.5}$	$\frac{1}{3.6}$	$\frac{1}{3}$	$\frac{1}{2.5}$	$\frac{1}{1.9}$
The supports level.	Max. moment in the 2 spans	0.0927	0.0919	0.0906	0.0897	0.0882	0.0868	0.0851	$0.0819ql^2$
	Max. moment over the centre pier	0.125	0.125	0.125	0.125	0.125	0.125	0.125	$0.125ql^2$
Centre supports depressed to most advantageous position.	Max. moment at centre support and in the two spans	0.1026	0.1019	0.1011	0.1004	0.0994	0.0983	0.0970	$0.0946ql^2$
	Depression in ft. of centre support = s .	0.0111	0.0164	0.0247	0.0357	0.0563	0.0815	0.1262	0.2468

TABLE II. (*for* 3 *Spans*).

Length of middle span in ft. $= l_2$.	$\frac{l}{l_2} =$		$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{9}{10}$	1
	$\frac{p}{q} = 1$	$s =$	$0.00042l_2$	$0.00119l_2$	$0.00148l_2$	$0.00171l_2$	$0.00205l_2$	$0.00182l_2$	$0.00092l_2$
	$\frac{3}{4}$	—	0.00039	0.00100	0.00123	0.00142	0.00169	0.00155	$0.00083l_2$
574	$\frac{2}{3}$	—	0.00037 .2123	0.00093 .5338	0.00114 .6543	0.00132 .7576	0.00153 .8782	0.00146 .8380	$0.00078l_2$ or .4477 ft.
328	$\frac{1}{2}$	—	0.00035 .1148	0.00082 .2690	0.00100 .3280	0.00115 .3772	0.00133 .4362	0.00132 .4329	$0.00072l_2$ or .2379 ft.
230	$\frac{2}{5}$	—	0.00033 .0759	0.00076 .1748	0.00092 .2116	0.00106 .2438	0.00123 .2829	0.00123 .2829	$0.00068l_2$ or .1564 ft.
148	$\frac{1}{3}$	—	0.00032 .0473	0.00072 .1065	0.00087 .1287	0.00100 .1480	0.00115 .1702	0.00119 .1761	$0.00064l_2$ or .0947 ft.
82	$\frac{1}{4}$	—	0.00031 .0254	0.00066 .0541	0.00080 .0656	0.00092 .0754	0.00106 .0869	0.00111 .0910	$0.00063l_2$ or .0517 ft.
60	$\frac{1}{5}$	—	0.00031 .0186	0.00064 .0384	0.00078 .0468	0.00089 .0534	0.00102 .0612	0.00108 .0648	$0.00063l_2$ or .0378 ft.
40	$\frac{1}{7}$	—	0.00030 .0120	0.00061 .0244	0.00073 .0292	0.00082 .0328	0.00099 .0396	0.00104 .0416	$0.00059l_2$ or .0236 ft.
25	$\frac{1}{9}$	—	0.00030 .0075	0.00059 .0147	0.00071 .0177	0.00079 .0197	0.00093 .0242	0.00101 .0252	$0.00053l_2$ or .0132 ft.

In these tables:

p = own weight per ft., q = live load per ft.,

l = span in ft., both spans equal in Table I., two outside ones equal in Table II.

l_2 = span in ft. of middle span in Table II.

The height of the truss is supposed to be $\frac{1}{10}$ of l and $\frac{1}{10}\frac{2l+l_2}{3}$ for Tables I. and II. respectively, with a strain of about 8500 lbs. per □″ in the chords.

Mr. Mohr, in his article, 1860, finds for a strain of about 8,400 lbs. per □ inch, two spans, height $\frac{1}{10}$ of span, modulus of elasticity = about 29 million lbs. per □ inch: $1+z=1+51\frac{y}{h}$, where z is percentage of strain in the chords at centre, due to lowering of centre support, y is amount of such lowering, and h is the height of girder, both expressed in same unit of length. We find, therefore, that for a lowering of the centre support equal to $\frac{1}{51}$ part of the height of the girder, the strain in the chord at the centre is already doubled.

Similarly, for three equal continuous spans, the same admirable writer finds $1+\frac{10\,z}{7-r}=1+69\frac{y}{h}$, where $r=\frac{\text{own weight}}{\text{live load}}$, each per unit of length. Such a girder would therefore require that the central supports should be only $\frac{1}{69}$ of the height of the girder lower than the outside ones, or that the bridge, when laid on its side and every member without strain, should have a crown at the points where it will rest on the centre supports, above a straight line connecting its two end-supported points of $\frac{1}{69}$ of the height of the truss, in order that it receive a strain in the chords over the centre supports of $2\times 8{,}400=16{,}800$ lbs. per □ inch. Only half the described deviation of level in the piers, taken in conjunction with half the described crown of the bridge, will produce a like effect.

For small bridges, especially, the measurements of level required to properly mount a continuous girder are practically unattainable,* and the advantage to be derived from lowering the centre supports is completely swallowed up in the disadvantages resulting from accidental or unknown changes of level. The most advantageous level for the central supports of a three-span continuous girder, as compared with three single spans, will result in a saving of some 18 — 30% of chord section; but an equal disadvantage arises from a variation of level in the points of support of $\frac{18}{27600}$ to $\frac{39}{27600}$, that is, for small bridges of $\frac{1}{1533}$ and for larger ones of $\frac{1}{708}$ of their own height.

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* It may not be out of place to call attention to the fact that this difficulty decreases for large spans, and again to note that it can always be entirely avoided by weighing the reactions instead of measuring the relative level of the supports.

An endeavor to calculate continuous girders, without having recourse to the measurement of the distances the several supports are "out of level," would therefore seem to be well warranted. The same remark is less true as regards omitting the consideration of variations in the moments of inertia of the chord sections. The time for this additional approximation will come, when we shall have obtained coefficients of correction from experiments with many kinds and qualities of finished bridges. It will be to these last, most nearly correct formulæ, that the coefficients will have to be applied. At present, the methods to which this paper has been limited can gain but little additional *accuracy*, by having regard to variations of the moments of inertia, so long as they contain the relatively equal *inaccuracy* of being without practical coefficients of correction. In the mean while it is gratifying to find that calculating without regard to moments of inertia of the chord sections is of the less importance the more nearly the actual sections are made exactly proportional to the strains in them. For the mathematical case, where the above proportion is exactly obtained, Mr. Mohr finds for two equal continuous spans, uniformly loaded: —

$M_2 = 0.146\ ql^2 =$ moment over the centre pier,

$A\ = 0.354\ ql =$ reactions at ends, and

$M' = 0.06215\ ql^2 =$ maximum moment in each span, when the change of moment of inertia is taken into account; and, as is well known,

$M_2 = 0.125\ ql^2$,

$A\ = 0.375\ ql$,

and $M' = 0.0703\ ql^2$, neglecting the same.

Similarly, for three equal continuous spans, uniformly loaded, the moment over the central piers, the reactions at the ends, and the maximum moments in the first and third and middle spans are found for the two cases of varying and of uniform moments of inertia: —

$$\left.\begin{aligned} M_2 &= 0.1067\ ql^2, \\ A &= 0.3933\ ql, \\ M' &= 0.0773\ ql^2, \\ M'' &= 0.0183\ ql^2, \end{aligned}\right\} \text{ and } \left\{\begin{aligned} M_2 &= 0.100\ ql^2, \\ A &= 0.400\ ql, \\ M' &= 0.080\ ql^2, \\ M'' &= 0.025\ ql^2.* \end{aligned}\right.$$

§ 4. LOADS AND REACTIONS OF CONTINUOUS DRAWBRIDGES. — Pivot drawbridges, when constructed as link-bridges, usually have *two* centre posts at equal distances each side of the pivot or centre. Indeed, whatever the construction of the girder, two supports at the centre will be the rule, and one only, exactly at the pivot, the exception. In the latter event, the formulæ for two equal and those for two unequal spans, for the several kinds of loading, will give the reactions for any case that may arise. It is proposed to discuss in full only the more difficult case of *two* centre supports, in the course of which the other and less usual case cannot fail to be likewise sufficiently elucidated, the more so as the two have many points in common. Take, first, the supposition of symmetrical loading only; that is, loading such that no tendency to pivot at the centre exists, and let us see the effect on the truss, of the several possible levels of the end supports as referred to that of the two centre ones. The former couple can always, and the latter two can for all cases of symmetrical loading, be considered as though on a level.

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* See also the article by J. W. Schwedler, 1862, p. 277, cited in Appendix.

Case I. The bridge open and loaded with snow. In Southern climates, and in those cases of railroad bridges that have no flooring, the snow load may of course be omitted. In this latitude (40° and above) and for highway bridges it cannot be overlooked. The end supports are wanting, the bridge being open, and we have a girder resting on two supports only; and the reactions at each must necessarily be equal to half its own weight, plus the weight of the snow on half its length.

Case II. The bridge shut and loaded with snow plus the maximum live load. It will be evident that the reactions at the four points of support depend now very much on their relative levels. If the end supports are wanting, — one extreme case, — the girder would be in the same condition as when open, as regards support, but subjected to the maximum loading; if the end supports were lifted so high as to lift the ends of the girder higher than the two central supports, by an amount exceeding or just equal to the deflection of the girder, acting as a single span-bridge between the two end supports when subjected to the moving load, — the other extreme case, — it would of course act as such a single span girder alone, and undergo a strain at the centre equal to what it would in *Case I.*, under an equal load and were there only *one* central support, but with a general reversal of the *directions* of all strains. Both these cases are, however, practically out of question, and exist only as logical possibilities; * but between them lie an infinity of supposable cases. Having eliminated from the discussion the consideration of measured differences of level of the supports, and substituted therefor that of the actual weights of the reactions, we next eliminate this latter by considering only the several cases when the end reactions are just equal to 0.* It will readily be seen, therefore, that for the case in hand, a bridge loaded with snow and a live load, the reactions at the ends may be made = 0: when the bridge is empty, when loaded with snow, or when loaded at its maximum, though this last brings us again to one of our suppositions that may practically be neglected. *Or any given number of tons or lbs. per unit of length or panel may be considered as acting on the girder continuously, and the balance will be supported at the central points, which is equivalent to saying that, when loaded with this balance, the skeleton outline of the girder will endure no reactions at the ends.* It is evident that, by thus dividing up the loading into two parts, one acting on the girder continuously, the other supported at the two central points alone, the total reactions at the four several points can in all cases be obtained.

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* The first is too outrageous a case of wrong for even an unconscientious draw-tender to permit; the other could be prevented by not furnishing said draw-tender with any apparatus to lift the bridge off its centre supports. In the case of railroad drawbridges whose ends are intended to be lifted, the passage of trains before such lifting has taken place may effectually

Another way to explain this process of finding the total reactions by first finding the reactions due to certain elements of the loading is this; and the remarks above, relative to what constitute "level supports," and what "supports out of level," speaking after the manner of the elastic line theory, must be borne in mind in this connection. Take, first, the girder loaded with any part of the total load q, and the end supports then brought just into contact with the ends of the girder. For all *future* loads, the reactions will be the same as those due a *straight* girder on level supports; in point of fact, it is a *curved* girder on supports just fitting it, which amounts to the same thing. Sometimes the strains in each *member* have been calculated in parts, due to different loads, and these then summed up; the plan submitted of summing up only the *reactions* in parts, and then getting the strains of members at one operation, is generally to be preferred.

A practical choice, for the present, will be to suppose the end reactions equal to 0 or end supports barely in contact, when the bridge is empty. It will shut a little hard in that case, when loaded with snow; but, as the draw is opened least about the time that heavy snow-storms occur, this will only be taking advantage of a convenient provision of nature, and not working

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be prevented by connecting the signal that the bridge is ready for travel with the bolt that bolts the draw ends into the fixed bridge, and then arranging this bolt so that it cannot be shut until the draw ends *are* lifted. Still, the ingenuity with which such labor-involving contrivances are sometimes circumvented is fully equal to that which produced them.

* Attention is here called to the three widely different cases of having a reaction = 0, a *plus* or a *minus* quantity.

against one. Certain drawbridges, as will appear, have the ends raised before being subjected to travel; but even for such it is proper, unless the draw-tenders are under rules no whit less strong than military rules, to allow for the case where travel goes upon the bridge *before* the draw-tender has lifted or sufficiently lifted the ends of the draw; in other words, always to take the reactions at the ends for the empty bridge just equal to 0 in calculating *Case II.* We shall have, then, the reactions at the ends due to a load equal to the live load of the bridge + snow load acting on a girder continuous over four supports, or to be derived from equations **51–76**, for the several kinds of loading and proportions of length of spans one to the other. To the centre reactions derived from these equations, add the weight of each half girder, to get the total reactions at the two central supports.

These two, *I.* and *II.*, constitute all the cases of symmetrical loading, and we next take up the more numerous cases of unsymmetrical loading. This leads at once to a discussion of the manner of support of the two central supports (of the nature of the turtle, that is under the elephant, that supports the world). We shall distinguish two sub-cases: (*a*) when the two central supports remain firm, the same as though they rested upon a solid, inelastic, stone pier, or as though the draw took bearing upon the circular girder, which in turn rests upon the "live ring"; and (*b*) where the live ring is not considered as offering support, and the whole weight is thrown, by suitable framing, upon the centre pivot.* It will be readily seen, also, that any deflection that may occur in this "framing" may be neglected: we have but to lower the end supports an amount equal to this deflection, to render it entirely nugatory.

For equal outside spans, which alone it will be necessary to discuss, (unequal spans are treated just like equal ones, only that the short span is loaded with concentrated loads sufficient to balance its deficiency of own weight), there will arise twelve different cases of unsymmetrical loading, belonging under (*a*) and (*b*) to three different styles of drawbridge, and according as the snow load is considered or not. We find, namely, that for unsymmetrical loading, load on Am (m is point at centre of bridge) whether sub (*a*) or (*b*), we very often get the reaction at D first $= 0$, then a minus quantity. That is, in the latter event, in order that the elastic line shall remain in contact with the end support opposite to the loaded side, the light end must be weighted or force of some sort must be applied to keep it in position. In the absence of such provision, it will rise or "kick up," especially if the loaded end is not very firmly supported. Now it requires but a little thought to see that so and so many tons, suddenly rising $\frac{1}{2}$ inch, 1 inch, or $1\frac{1}{4}$ inches, &c., many times a day, and as suddenly falling again, and with nothing else to spend their work upon, are only so many foot-pounds assiduously laboring to destroy the bridge or the piers. Such a structure partakes too much of the nature of a tilt-hammer to be entirely satisfactory as a permanent drawbridge. Upon taking up therefore the question as to how to prevent this tipping, we find two methods in practice, the first long in use, the second apparently for the first time used by Mr. Charles Macdonald of New York. They may be likened to the two methods at command for preventing a horse or an ox or a man from kicking. One way would be to fasten his heels to the floor, the other to lift them up so high that any attempt to kick would indeed diminish the strain upon the chords that held said heels up, but would never result in their going any higher. In the same way may be managed this restive end of unsymmetrically loaded drawbridges: it may either be lifted up so high that the reaction D shall under no case of loading become either equal to 0 or a minus quantity, or it may be fastened down, this last being Mr. Macdonald's way.

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* In some drawbridges the centre pivot is used only as an axis of rotation, never as a support; in such, of course, the cases (*b*) cannot arise.

As previously remarked, and as often found, the ends may also be left free; and we have therefore the above-stated twelve cases (two of each kind, according as we consider the snow load or not) of unsymmetrical loading as follows:—

Case	*III.* (*a*)	ends	free,	on	firm	central	supports.
,,	*III.* (*b*)	,,	,,	,,	pivoted	,,	,,
,,	*IV.* (*a*)	,,	latched,	,,	firm	,,	,,
,,	*IV.* (*b*)	,,	,,	,,	pivoted	,,	,,
,,	*V.* (*a*)	,,	lifted,	,,	firm	,,	,,
,,	*V.* (*b*)	,,	,,	,,	pivoted	,,	,,

Case III. (*a*). Let us again so place the end supports that when the bridge is empty they shall be just in contact with the bridge, no more and no less; that is, their reactions shall equal 0. Imagine now a girder so supported, no part latched down, to be loaded on length Am (m being point at centre) with dead $+$ snow $+$ live, and on length mD with dead $+$ snow. This will generally (it may be proven in each case) give the maximum *absolute* strains in the chords at least. It would be *relatively* more severe for the two main spans of the girder, to suppose one side loaded with dead $+$ snow $+$ live, and the other with dead only; but there is a little of the absurd in thus supposing a bridge only half covered with snow, and at the same time it does not furnish absolute maxima. We take therefore, in all the unsymmetrical cases of loading, for maximum strains, dead + snow + live on Am, and dead + snow on mD. The snow load has, however, an important effect in forming the nature of the end reactions and of some of the webbing in some cases; and it is necessary carefully to consider the bridge with and without snow upon it, as it may easily happen that a bridge that is firm under snow + live on one side, and snow on the other, becomes tipping and unsatisfactory with the snow load removed. To determine the nature or positions of the end supports and in some cases certain members of the webbing, we therefore calculate all the unsymmetrical cases for dead + live on one half and dead only on the other, as well as for snow added to each. In the case of panel bridges having only one panel over the centre, half the centre panel load can be supposed to be applied at the inner extremities each of spans AB and CD : if there should be more panels, the formulæ will easily admit of being extended to cover those cases also; and, for plate girders and uniformly distributed loads, the load on centre span can likewise, if desired, be taken into account, so as to make in all the unsymmetrical cases the load on just half the length of the girder.

To resume, then, the special case under consideration, we have a girder on four supports, unequally loaded (Fig. 2), and B and C higher than A and D by an amount just equal to the deflection of the bridge under its own weight when resting on B and C alone, no matter what the exact depth of such deflection be in decimals of a foot or inches.

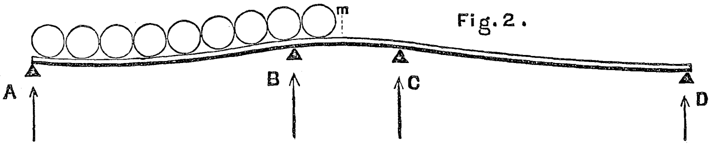

Fig. 2.

First, take the action of the bridge under its dead load only. Reactions at B and C are each equal to half of dead on whole girder. Now place end supports in contact. For all *future* loadings the bridge is equivalent to a straight girder on level piers. Add snow + live on Am and snow only on mD. Find reactions for this unsymmetrical load from **63 – 66** (or **69 – 72**), four supports on a level, and to B and C previously found add (algebraically) the new B and C for the final B and C. A and D are found at once and directly. Now see whether C and D, one or both, are plus or minus. We can have both plus (a),* C minus and D plus (β), (D minus and *C* plus is impossible), and both minus (γ). If (α), the above calculation was correct; if (β), it is a case of two unequal spans *AB* and *BD*. Start anew (Fig. 3) this time with the whole of the dead load of the whole girder on *B*.

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* Equivalent, if D is large enough, to *Case V.* (a).

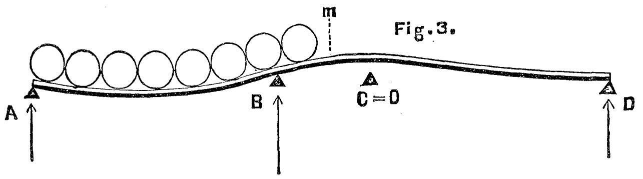

Fig. 3.

But, if put in that position, it would need still another force or better couple to keep it there, since *BD* is longer than *AB*, and the dead load is supposed to be symmetrical about *m*. Such a balancing couple could be applied at *A* downwards like a load, and at *B* upwards like a reaction, — that is, *diminishing* any *A* to be found hereafter, and as much *increasing B;* or at *D* upwards and *B* downwards, — that is, *increasing* any *D* to be found hereafter, and as much *diminishing B* ; or finally at both *A* and *D*, with the same respective characteristics. Such force is without doubt applied at both *A* and *D;* and, to find it, we must first see how much of the unsymmetrical load on *Am*, when the supports are as in *III.* (*a*) (*α*), will just make $C = 0$, which can be done from **63 – 66**. For this much of the unsymmetrical load and dead + snow load, the reactions are as in *III.* (*a*) (*α*); for the balance of the unsymmetrical load, they are according to **32 – 34** (or **43 – 45**), for two unequal spans *AB* and *BD*. This is evidently a rare case, and moreover moves between very narrow limits of possibility. It would require a proportionally very large live load, and peculiar ratios of loads and lengths, to produce such a case: it is given, however, for the sake of completeness, and well illustrates the general adaptability of the method of separating the effect of combinations of loads into the effects of certain elements of the same.

Case III. (*a*) (γ). If both *C* and *D* are minus (Fig. 4), it is a case of one span and an overhanging arm *BD*. We now have again the whole of the dead load upon *B*, and the reactions required to balance it about *B* can be applied at *A* and *B* only. These will tend to diminish any future *A*, and

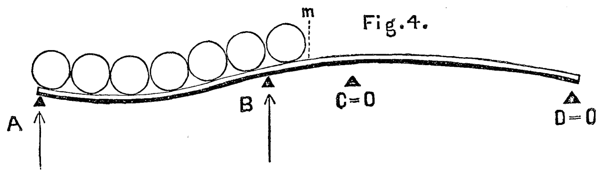

Fig. 4.

will equally increase *B*. The snow will now act precisely as does the dead, and the live will be distributed between *A* and *B* according to the simple and well-known laws of the lever. Necessarily *A* and *B* are the only reactions. This, again, is a rare case, and requires still greater live loads in proportion to the dead load and snow. Equations to meet *III.* (*a*) (β and γ) could be evolved by introducing the conditions *C*, or *C* and *D*, $= 0$, and the known values of moments that that implies, in the equations of the elastic line. See book of Weyrauch, often cited.

Case III. (*b*).—Fig. 5. The end supports are so placed, that for the empty bridge, they shall again be just in contact: dead + snow + live on *Am;* dead + snow on *mD*. For the dead load, *B* and *C* again each equal half dead load of whole girder. Now find *A*, *B*, *C*, and *D* from **89 – 91** for live on *Am*, nothing on *mD;* also, *A*, *B*, *C*, and *D* for snow over whole bridge from **63 – 66** (four supports on a level).* *C* cannot now be a minus quantity: it must equal *B*, in order that there shall be equilibrium, and there remains only to see whether *D* is or not. If plus (*a*), the reactions found are correct; if minus (β) (Fig. 6), take first again *B* and *C* each equal half dead load of girder. Then, snow over whole girder from **63 – 66** (four level supports), then find (from **89 – 91**) what part of live will just make *D* equal to 0. The reactions for that part are found from **89 – 91**; those for the balance of live from **93 – 94**. Or, a shorter way, take first dead on *B* and *C*. If now, in such a pivoted girder, forces ΣP on *AB*

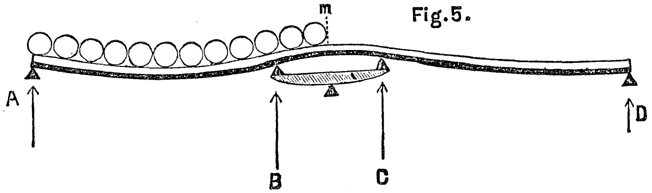
Fig. 5.

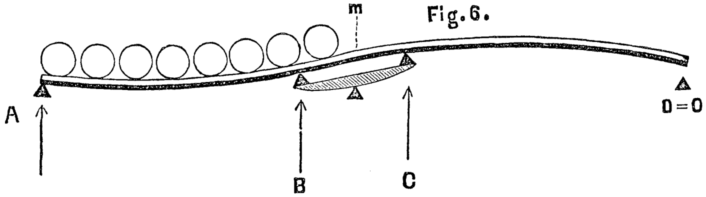
Fig. 6.

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* The two operations just described with different formulæ could be done in one operation and by only one set of formulæ by introducing into **89 — 91** the terms for loads on *mD* as well as on *Am* only.

cause D to become $=0$, that part of the symmetrically distributed load which is on CD will just balance the part on AB, about the pivot under B and C, which will leave the ΣP acting only according to **93-94** on A, B, and C, and the symmetrical loads must all rest in equal parts on B and C.

Case IV. (*a*).—Fig. 7. The end supports are so placed that for the empty bridge they are just in contact. Centre supports firm. Ends latched down. Loading as before. We have, first, half of dead on B and C respectively. From **63-66** find A, B, C, and D for snow + live on Am, snow only on mD, and add middle terms to B and C already found. It matters not now whether the resultant D be plus, minus, or 0 : in either event there is a body at hand to produce the needed reaction. If C is plus (a), the reactions are correct; if C is minus (β), Fig. 8, it is again a case of two unequal spans, AB and BD. Start again with B and C each equal to half of dead on whole girder. Find and sum up A, B, C, and D for snow on whole girder, from **63-66**; now find what part of live on Am will make $C=0$ from **63-66**; that part will produce reactions, also to be found from **63-66**, the balance will act on A and B only, according to equations **32-34**, for loads on two unequal spans. This also may be considered as a rare case in actual practice. The minus values of D will show the necessary strength of the latching apparatus.*

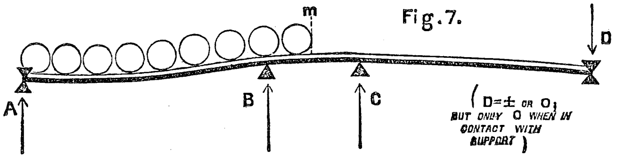
Fig. 7.

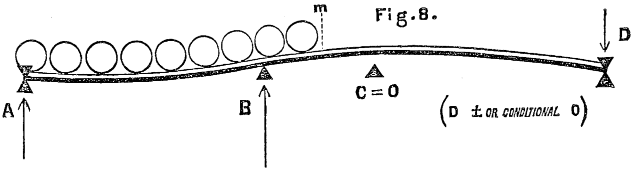
Fig. 8.

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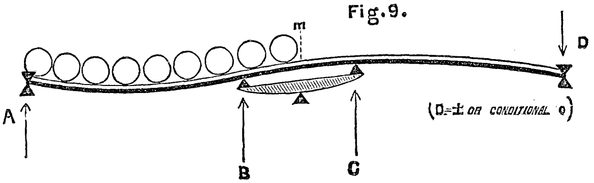
* The latching apparatus must be constructed to resist, also, a strain that will arise, due to a difference of temperature in the two chords, tending to raise the bridge ends, *i.e.* top chord warmer than bottom chord. See C. Shaler Smith's paper cited below. The effect of general changes of temperature on the strains and shapes of trusses is very lucidly treated in Ritter's book, cited in the appendix; and a study of the principles of the same would, no doubt, soon lead to the method to be followed in computing the strains and changes produced by a difference of temperature between the two chords of a bridge.

Case IV. (*b*).—Fig. 9. The end supports as before. Centre supports pivoted. Ends latched down. Loads as before. We have, first, half of dead on B and C respectively; then, for snow on girder, A, B, C, and D,

Fig. 9.



to be found from **63–66**; also live on Am, with nothing on mD, A, B, C, and D, to be found from **89–91**; sum up, and we have the final A, B, C, and D.

The relative merits of *IV.* and *V.* it is proposed to discuss further on.

Case V. (*a*). End supports on such a level, relatively to B and C, that they are more than high enough to prevent either A or D from ever being $= 0$. Centre supports firm. Loading as hitherto. Now as to the amount the ends should be lifted. The only object of lifting them can be either to prevent what we have called the "kicking" of the light end or to regulate the strains over the centre. We have to deal with the first only at this time. Find, therefore, with what loading of the whole girder the light end reaction is just equal to 0, when the heavy side is loaded with its maximum moving or live load, and then make the end reactions $= 0$ at a loading *greater* than this, so as to attain a certain factor of safety against such an occurrence. This subject is treated of, for both the case of the pivoted draw and of two unequal spans, in §§ 13 and 14 of Part II. To illustrate, in the numerical example *V.* (*b*), if the ends are raised (see § 13), so that 2.24 tons* per panel react on A, B, C, and D, as a girder continuous over four supports, with B and C pivoted, it will make D equal to 0, when Am is in addition loaded with 12.6 tons per panel. In such a case we would take, say 3 tons per panel, as the uniform load, of which the end supports must carry their due share, in order that the live load on Am shall never cause D to equal 0. And it may be noted here that railroad bridges, subject to having trains move over them at a high rate of speed, would need a larger factor of safety against this occurrence than roadway bridges.

In our calculations we therefore take the end reactions equal to 0, when the skeleton lines of the girder are loaded with the own weight, minus the weight selected as above, which we will call *s* (for stability), and proceed.* Just as *s* must now always make *D* plus, it can and should be required to always keep *C* plus. The case is therefore analogous to *III.* (*a*) (α), only substituting dead — *s* for dead, and snow + live + *s* for snow + live, and similarly.

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* Wherever the word "ton" is used in this paper, it means 2,000 lbs.

Case V. (*b*). End supports lifted a certain amount higher than for *I.–IV.* Centre supports pivoted. Loading as hitherto. *s* is found from § 12, Part II., which reduces this case to *III.* (*b*) (α). Or find *A*, *B*, *C*, and *D* from 63–66 (two central supports pivoted), for snow + live + *s* on *Am* and snow + *s* on *mD*. *D* will surely be plus, if *s* has been taken right, and after adding half (dead — *s*) on whole girder, to *B* and *C* each, we have the final *A*, *B*, *C*, and *D*.†

§ 5. PRACTICAL CONSIDERATIONS IN SELECTING THE KIND OF SUPPORTS FOR DRAWBRIDGES. — The subject, as left in the last section, is perhaps seemingly complex, but seemingly only. We had in all: *I.*, *II.*, *III.* (*a*) (α, β, & γ), III. (*b*) (α & β), IV. (*a*) (α & β), IV. (*b*), V. (*a*) (α & β), and V. (*b*), — that is, fourteen cases, or, adding those without snow, twelve more, in all twenty-six cases; but this is only because of the generality of the treatment of the subject, which adapts it to *all* cases of double centre post drawbridges that may occur. All those marked *III.*, in all ten cases, need never, and it is advised should not, occur, or at least reduce to *V.*; that is, have *D* equal plus. For any particular loading and form of truss, only one of α, β, or γ can ever prevail, and the choice of end support determines whether to use *IV.* or *V.* For any one drawbridge, therefore, we shall have, generally, only four cases of loading to consider; and if it is desired to calculate for *both*, firm centre supports, and for the two central supports pivoted, this will add two more cases, making in all six.

To calculate for partial loadings of any one span will not be necessary for the chords. This can be proved mathematically: numerical proofs can be derived from the equations given, for the construction of the formulæ render the introduction of a single load at any point or distributed loads of various values per unit over any length in any position an easy matter. Instead of ΣP substitute for distributed loads $\int qdx$ between the limits that the load q is distributed. In case of the webbing, it may occasionally be necessary to consider the spans as partially loaded.

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* *s* may be taken 0, or even a minus quantity, as far as tipping is concerned, if the dead load is so great, in comparison to the live, and the length of spans are so proportioned, that the dead alone prevents *D* from being equal to 0. This weight *s* should also be such that the raising of the ends due to it will make *D* plus, when the draw chords are unequally affected by temperature, and the draw ends tend to rise on that account, that is, where the bottom chord is warmer than top chord. See C. Shaler Smith's paper cited below.

† It is proper to remark that the snow load, or, more exactly, the fact that the snow load reactions are not equal to 0, when the bridge is shut, have introduced a triple element into all the unsymmetrical loadings. By leaving out the snow load, as in the second series of cases of unsymmetrical loading, or by making the reactions equal 0 with the snow load on, the several deductions are simplified.

Generally speaking, *Case I.* rules the strains at each side of a point about $\frac{1}{3}$ the length of the two end spans, measured in either direction from the centre supports; *Case II.* gives maximum strains over the centre; and the unsymmetrical cases rule the dimensions of the two main spans, from where *Case I.* leaves off to the two ends respectively, and the strength of the latching or position of end supports in *IV.* and *V.* There may be some question whether it is wise to make the end reactions $= 0$ for the empty bridge in *Case II.*, the reason for which has been explained in discussing that case. By making them due to s, or still more, for *Case II.*, we decrease the strains over the centre, and, if certainty existed that the bridge would, before loading, be always supported properly at either end, and *Case II.* were then calculated for reactions at ends due to s, *Case V.*, which follows from the same arrangement, would have an advantage over *Case IV.* in producing less chord and web over the centre. If this certainty does not exist, and *II.* be calculated for end reactions equal to 0, *IV.* and *V.* are placed on an equal footing, as far as centre sections are concerned. A great point in favor of *IV.* is the little time and power required to latch the ends, as compared with that needed to lift them; though this last should not be overestimated. The power required to lift grows from 0, and can increase as high as half the weight of the girder, if the bridge be lifted off its centre supports.

We shall see, however, in a numerical example, that in point of fact comparatively little, sometimes no lift, is required to keep the bridge steady under the unsymmetrical load, according as the centre supports are pivoted or firm and snow on or off. In the numerical example, when pivoted and with snow off, it is necessary to have the end reactions due to a distributed load of about 4.5 tons per panel, or, roughly calculating for a 200 ft. draw end to end, about $\frac{3}{8}$ of 25. tons, or about 9.3 tons, and that only at the *end* of the lift.* By means of eccentrics, toggle-joints, or any other of the mechanisms that have *increasing* power, such a final pressure is easily produced. Such an increasing and rising pressure can also be produced by a *constant*, descending counter-weight. See "Zeitschrift für Bauwesen,"

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* In the numerical calculations, $s = 4.8$ per panel, and $A = D$, when bridge is empty $= 2.9520 + 1.7712 + 3.1980 + 1.9188 = 9.84$. See table of reactions.

Berlin, 1871, plates 41 and 42, or also "Engineering," Vol. 12 (1871), page 154.*

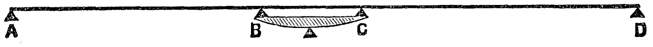
In bridges subject to frequent opening and shutting, whatever arrangement is used at the ends, latch or lift, should be operated from the centre of the drawbridge to save time.

If *IV.* is to be used for railroad bridges, it must be carefully constructed, and the bolt that serves to latch the draw should be made to fit as exactly as possible in its socket, or all "lost motion" be taken up by the mechanism; if there be any play-room above or below, it would allow of appreciable destructive power being exercised respectively at the light end or over the centre every time an engine runs on the bridge.

A description of Mr. Macdonald's drawbridge at Point Street, Providence, R. I., 250 ft. long, and fitted with a latching apparatus, may be found in "Engineering," March 21, 1873, p. 202.

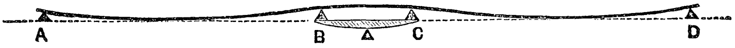
Whether the centre supports should, in the calculation, be considered as firm or pivoted (*a* or *b*), or the calculation made for both, the construction of the bridge must determine. Drawbridges will be differently constructed in this respect and in that of the manner of end supports, among other reasons, accordingly as they are more or less frequently opened. It would be an approximation, when the draw rested on the pivot, to consider it in effect as converted into a girder of two spans only, the pivot forming the middle support, and the framing over the pivot part of the truss, — but an approximation only, as may be proven. In a case of *symmetrical* loading of *AB* and *CD*, for example, (Fig. 10) the truss is evidently a case of *three* continuous spans. If, now, the reactions *A* and *D* are calculated, they will be found *less* than if calculated supposing the truss to consist of *two* spans only, with exactly the same loading.

Fig. 10.



And the reason for this is not far to seek. The elastic line would take the shape as drawn in Fig. 11, rising higher than the level of the supports

Fig. 11.



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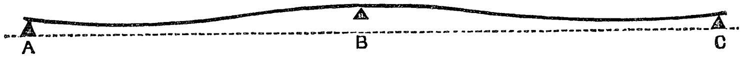
* The same apparatus, together with the peculiar, and in some situations admirable, kind of drawbridge to which it is in this instance applied, is fully and clearly described and calculated by Häseler, Civil Engineer, in "Zeitschrift des Archt. and Ing. Vereins zu Hannover," 1869, p. 412. The form of draw referred to is made without any "live ring," and the pivot is used as a support only when the bridge is being opened.

in the middle, or the case would be equivalent to the one of two spans, as sketched in Fig. 12, where the centre support is *higher* than the end supports;

Fig. 12.



but this, we *know*, decreases the end reactions, showing why, for a girder of three spans, the end reactions are less than for the same girder considered as composed of only two spans, but with the three supports on a level. Both cannot be right, and from the above three spans evidently is. Making the end reactions less involves making the centre ones greater, and the demonstration just given is as applicable for unsymmetrical loads as it is for symmetrical. To convert the girder into one of two spans only, it would be necessary to give it only one centre post, or arrangement of struts equivalent thereto, these bearing on the centre pivot, or on a *single* arm framing resting upon the same.

§ 6. Strains in the Girders.—It may seem strange to some readers that no word has yet been said about any thing but finding the *reactions* under the girder, as though that constituted the whole problem of the calculation of the strains in continuous drawbridges; and with but little exception it does. The reactions once found, any structure, of just the proper degree of strength to meet them and the loads that caused them, can be interpolated between the two, without the slightest difficulty or labor other than that inherent to the calculation of any framed structure, if the right way be only followed. And, as this has been written for the purpose of contributing to the elucidation of the construction, more especially, of *panel* drawbridges, the writer ventures to remark that he shall not consider his labors to have been in vain if, achieving nothing else, he will awaken in the minds of some of his readers a realizing sense of the beauty—that seems to be the correct appellation to apply to them—of the principles used by Aug. Ritter* in the calculation of all link structures, and which it is now intended to describe.

To understand this thoroughly, let us return to first principles and see what a framed link structure really is. It may be described as a properly designed skeleton system of lines, each line endowed with the faculty of exercising muscular or elastic force in both directions, each line pivoted at its extremities, and then acted upon in *space* by generally only vertical forces, or, including roof-trusses, by vertical and horizontal forces, —in short, by forces in the plane of the structure; these forces are, in the first case, the reactions and loads, in the second the reactions, loads, and force of the wind. We have given the skeleton lines, the loads, and points of application, and the points of application of the reactions. We must find what has been called above the muscular force required to keep every thing in equilibrium, and the only way, in cases of continuous girders, to do this, is by first finding the reactions; and, going farther, there is much in favor of the statement that, when on account of there being only two supports, as in case of single spans, the laws of the lever (which, however, are only a special case of the elastic line) suffice, and strains, as this muscular force is more frequently called, can be found without starting from the reactions, the latter method is the best, simplest, and easiest remembered.

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* See Ritter's book, cited in Appendix. In Van Nostrand's "Eclectic Engineering Magazine" for 1871, pp. 136 and 332, Ritter's method may be found translated and described for two examples, in a way, however, that to the author seems devoid of appropriate emphasis.

Taking now a girder as above described, acted upon by its forces (loads and reactions), and considering all parts in equilibrium, we have in any section:—

1. The sum of all the horizontal components each side the section must $= 0$, or $\Sigma H = 0$.

2. The sum of all the vertical components each side the section must $= 0$, or $\Sigma V = 0$.

3. The sum of all tendencies to rotate about any point in the plane of the girder — that is, all moments — must $= 0$, or $\Sigma M = 0$.

Ritter's simplification consists in using equation 3 *only*, which can always be done by merely *so choosing the points of rotation* that they shall be situated at the intersection of the direction of two out of three forces which are being investigated. The moments of two thus become 0, and there is left a simple equation for the value of the third. The general enunciation of the principle is this: Conceive a section of the girder, taken in such manner that it shall cut, if possible, only three members, and imagine the forces X, Y, and Z applied at the section, in the line of these members, and representing the strains in them. For that part of the structure which lies at either side of the section write the equation of moments, and so choose the point of rotation that, in determining X, the point of intersection of the directions Y and Z is taken; in determining Y, that of X and Z, and in determining Z that of X and Y.

For example, take a roof truss (Fig. 13):—

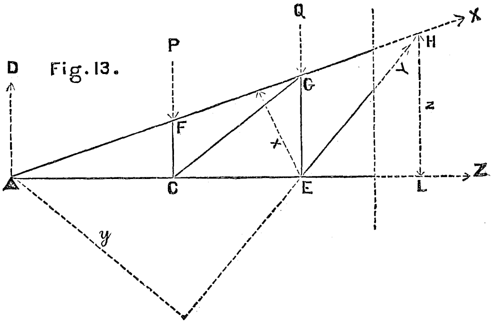

Fig. 13.

$D =$ reaction, P and Q are loads, X, Y, Z strains required, x, y, and z lever arms. Moments with the motion of the hands of a clock, $+$; against it, $-$.

1. About E, intersection of Z and Y,

$$Xx - P.\overline{CE} + D.\overline{AE} = 0,$$

$$\text{or } X = \frac{P.\overline{CE} - D.\overline{AE}}{x}.$$

2. About A, intersection of X and Z,

$$-Yy + P.\overline{AC} + Q.\overline{AE} = 0,$$

$$\text{or } Y = \frac{P.\overline{AC} + Q.\overline{AE}}{y}.$$

3. About H, intersection of X and Y,

$$-Zz - Q\ \overline{EL} - P.\overline{CL} + D.\overline{AL} = 0,$$

$$\text{or } Z = \frac{D.\overline{AL} - Q.\overline{EL} - P.\overline{CL}}{z}.$$

Where a member cannot be cut by a section cutting only two others, it will always be possible (unless there is a redundance of members, and that means indeterminate strains and bad construction) to cut, if there are four together, some one of them, if five, two of them, &c., by a section taking in only three members; and, their strains once determined, they can be used as known forces in getting the others of the same group. Or, any number all meeting in one point and one more, may be cut, and the strain of this odd one determined; it being remembered that the section may be taken in any direction, straight or curvilinear. For trusses with parallel chords the demonstration is still correct; we have but to introduce ∞ as the distance at which the chords meet, and it will be found that the ∞s all cancel out, leaving easily remembered equations and forms of equations.

This constitutes, in fact, the whole method, which needs but to be tried to be appreciated. Its advantages are, primarily, that it throws aside all burdens upon the memory, in the shape of special rules for special cases, rules for each different kind of roof or bridge truss, not to mention a set of rules for variations of one and the same kind of girder.* The advantage of this it is difficult to overestimate; it exceeds, if any thing, the value of the well-known Napier's rules in Spherical Trigonometry. Another value lies in the readiness with which the strain on any one member can be calculated independently from that on any other; and, finally, in simple trusses, in being able to recognize at a glance, from the form of the equation for the strain on each member, what loads diminish and what loads increase the same; that is, under what loading it is maximum or minimum. The numerical examples have all been worked according to Ritter, and the reader is referred to them (and more especially to Ritter's book) for more on this branch of the subject. In the cases of finding the strains on the diagonals, it is frequently simpler to choose the first of the equations of equilibrium written above, instead of Ritter's method; that is, after having found the chord strains, by remembering that the horizontal components must equal 0, the diagonal strain becomes merely the horizontal component of the strain in one chord minus that of the strain in the other, multiplied by the secant of the angle the diagonal that connects them makes with the horizon, and it is of the kind of strain, tension or compression, according as needed for equilibrium. If the two chords are unlike, it always has the same quality, *t.* or *c.*, as the smallest; if both chords are alike (happens in Warren girder), the quality is opposite to that of the chords, and must equal their *sum* instead of difference.

Plate girders are not specially treated of in this paper; nothing new is to be said of their calculation, and, by getting equations for moments from the reactions, the curves of moments (and lines of shear from reactions) are easily drawn, and calculations of sizes of parts made as well known and long used. The value of plate girders for drawbridges is seriously diminished by the large surface they expose to the force of the wind. Cases are not wanting, where such bridges have been blown off the centre pier or have shut with such force as to break the centre pivot, &c. For large spans especially, their use must be decidedly condemned.

§ 7. PRACTICAL CONSIDERATIONS AS TO CHOICE OF FORM AND SYSTEM OF TRUSS, TURN-TABLES, AND CENTRE PIVOTS; AND IN CONCLUSION.—

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* Most of these rules are, moreover, of the perplexing kind, sometimes known as "sink or swim" rules; *i.e.*, the answer sought is found in one of two ways, but you cannot tell which.

Laissle and Schuebler, in their work of 1870, give at the close a digest of 16 typical iron bridges that were built between 1855 and 1870, of all conceivable shapes and forms, both as regards chord, web, position of platform, number of trusses and corresponding number of tracks; two of them continuous over three spans; in fact no two alike, reducing them all to the same strain per unit of section, under the same load per unit of length. The result, if intended as an exhibition of the superiority of one style over another, is entirely nugatory, with the single exception that it is clearly better to support two tracks on two trusses only, than it is to support them on three or four. The authors arrive at the conclusion that the economy of each bridge depends, not on the choice of any one from among the dozens (hundreds it might almost be said) of good styles of skeleton outlines of chord and web that are fit to be used in any given locality, but on the skill of the constructor in each particular case; but this again is resolved, almost wholly, into that of reducing to a minimum that part of the weight of the bridge which is literally "dead" weight, — that is, carries no load, does not exercise muscular energy, and is only so much apoplectic, adipose tissue. Again, every member necessarily brings with it into the bridge more or less of this very dead weight; the less members, therefore, other things being equal, the better the bridge, the more load it will carry with the same total weight of material in it, or, conversely, the less material it will require to carry a given load. The rules for a choice of system for drawbridges will not be different from those for single fixed spans, with some few exceptions due to reversal of strains from *c.* to *t.*, and *vice versa.*

At first sight, it might seem as though the height of the truss should be in a very different proportion to the span in draw from what it is in fixed bridges; because, when the draw is open, the span, of which each side represents half, is in fact double that of the doubly supported single span, of equal moment. It must not be forgotten, however, that when it is in this position the bridge is not loaded. Taken altogether, therefore, about the same proportions of height to span will obtain in continuous as in single spans, though with a leaning towards greater height in drawbridge spans than in single spans of equal length. In a recent competition for a draw 200 ft. over all and for two fixed spans each 121 ft. long, the same designer made only $660 difference in favor of 21 ft. trusses as against trusses 15 ft. high, on a bid for the latter height of $50,560. Perhaps Mr. Baker's* investigation as to the proper height of single span trusses and plate girders could be extended to the consideration of continuous spans also.

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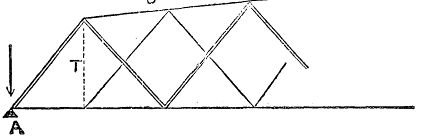
* On the Strengths of Beams, Columns, and Arches. B. Baker, London. E. and F. N. Spon, 1870.

In continuous drawbridges, more than in single spans, the defects of separate counter-bracing become apparent. It is very much a question with the writer, whether separate counter braces and ties have not been used far too often in all structures, wooden or iron, but especially in the latter. A member that will resist both tension and compression can nearly always be made to weigh less than one to resist compression or tension running in one direction, added to its counter of like quality running opposite to it, especially when we have regard in the two cases to the useless dead weight above spoken of. In the way indicated, — that of introducing members capable of resisting both *t.* and *c.*, wherever they are liable to exchange of strain, — *any* kind of truss can be made to dispense with the customary counters and their extra weight.* This is recommended in all continuous spans, drawbridges included. In case of wooden trusses also, an iron rod parallel and beside the wooden strut, so as to make a counter-tie instead of counter-brace, would probably make a good wooden continuous truss.

A point to be careful about also is that of either superfluous or deficient members. The numerical example is one sent in in a recent competition for a drawbridge, slightly altered. As first designed, the end panels were as in Fig. 14, the dotted line representing a tie rod. Now when such a truss is latched down, which is equivalent to putting a concentrated load on *A*, it will exert compression on the piece *T*. *T* cannot take, however, even the compression due to the loading on its own system, being only a rod, and should be made so as not to suffer from such compression, by having a link in it, or by being passed loosely through the top or bottom chord, &c. But, when thus cleared of compression, the minus reaction at *A*, which should

Fig. 14.



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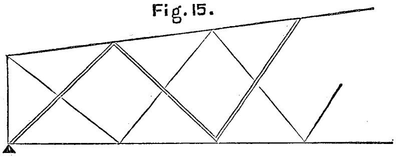
* It is curious sometimes to see counters introduced where, in point of fact, they appear like "cats in a strange garret." A "Post" truss, for example, is nothing but a Warren girder whose struts and ties are at an unequal angle with the horizon; and this web, instead of being made, like all other Warren girders, to take either *t.* or *c.* at the centre of the truss, is fitted out with counters in the shape of tension rods. They are, however, more proper here than in some other cases, owing to the great length of the regular tension members, which without them would have to be made to resist compression also.

pervade the whole webbing, can act on one system only, the one drawn in double lines, which must be characterized as undesirable. For this reason the end has been changed as in Fig. 15. Each system can now take any load or reaction due to its own loading, and the end post belongs to the second system, the one drawn in *single* lines.*

Fig. 15.



A valuable paper on turn-tables and pivot centres by C. Shaler Smith will be found in the July number, 1874, of the "Transactions of the American Society of Civil Engineers." The data given there are probably unexcelled by those to be found scattered in the descriptions of various drawbridges. It is more such data that the profession is sadly in need of.

In the same number, Mr. Macdonald cites an equation from the ("Journal of the Society of German Engineers") Zeitschrift des Vereins Deutscher Ingenieure, Berlin, April, 1874, for the safe pressure on bearing rollers. The same subject may also be found analytically treated by Jules Gaudard, in "Études comparatives de Divers Systèmes de Ponts en fer," quoted in Spon's Dictionary of Civil Engineering, p. 789, and in Winkler's book, cited in the Appendix.

The writer makes no apology for the length of this paper or of any formulæ to be found in it. There is no "royal road" to the calculation of the strains in continuous girders, and the methods described in this paper are believed to be an improvement upon and are shorter than any of the exact modes of calculation that have come under the author's notice. It has been to him a pleasant task, though task it was. But there is no branch of engineering so cleanly cut and finely moulded in its results as this of finding the strains in framed structures. The reactions must first be equal in algebraic sum to that of the total loads; then, around any point of meeting of the several members in the structure the vertical components *up* must just balance those *down;* those to the *right* must balance those to the *left;* and any member cut in two must reveal in its interior an elastic or muscular force that will prevent revolution about *that* point around which the parts of the structure *would* rotate, but for the duty by it performed. And the members and reactions must do all this, if need be, to any desired place of decimals, just as in the ledgers of some stately mercantile house the separate accounts and the trial balances must all foot up equally on both sides to the fraction of a cent. The principles involved in the one case are no more certain to produce the desired results than those upon which hinges the other; and, if those clear results are not forthcoming, it is but a case of human error or imperfection. The *principles* remain, perfect, immutable, and eternal.

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* There is still in this arrangement *some* indeterminateness; viz., the absolute direction that the reaction at A will take, whether up the end post or the first strut or what part on each. This is a defect common to all multiple systems of web, but is not of serious importance.

PART II.

§ 8. INTRODUCTORY TO THE MATHEMATICAL INVESTIGATIONS. — The arrangement of this Essay is intended to be such that, while the argument is all to be found in Part I., and its application illustrated in Part III., Part II. shall contain only the mathematical demonstrations, or the derivation of the formulæ presented for use, and used in Parts I. and III. The reader that is prepared to accept these formulæ on faith, therefore, need not read Part II. at all, but will, it is expected, find in the remainder of the paper sufficient to enable him to design any revolving drawbridge. Those that seek for mathematical proof will find the basis of Part II. in the work of Weyrauch, to which reference has many times been made ; and the equations, taken from it, that are needed for the development of the formulæ it is now proposed to find, are the following :—

17. $$M_{r-1}\,l_{r-1} + 2\,M_r\,(l_{r-1}+l_r) + M_{r+1}\,l_r = -\left[\frac{c_{r-1}-c_r}{l_{r-1}} + \frac{c_{r+1}-c_r}{l_r}\right] 6\,EI - \frac{1}{l_{r-1}}\sum_{r-1} Pa\,(l-a)\,(l+a) - \frac{1}{l_r}\sum_{r} Pa\,(l-a)\,(2\,l-a).$$

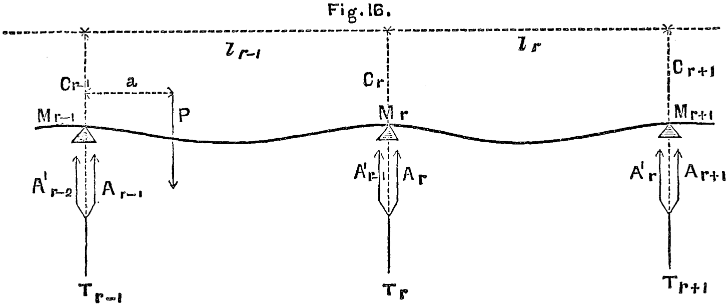
17*a*. $$M_{r-1}\,l_{r-1} + 2\,M_r\,(l_{r-1}+l_r) + M_{r+1}\,l_r = -\left[\frac{c_{r-1}-c_r}{l_{r-1}} + \frac{c_{r+1}-c_r}{l_r}\right] 6\,EI - \tfrac{1}{4}\,q_{r-1}\,l^3_{r-1} - \tfrac{1}{4}\,q_r\,l_r^3.$$

20. $$-V_0 = A = \frac{1}{l}\left[M' - M + \overset{1}{\Sigma}\,P\,(l-a)\right], \text{ and}$$

21. $$V_l = A' = \frac{1}{l}\left[M - M' + \overset{1}{\Sigma}\,Pa\right],$$

in which the several characters have the meanings indicated by Fig. 16, and the schedule given below.

Fig. 16.



M_r, M_{r+1} &c., are moments over the supports, at left hand of the spans l_r, l_{r+1} &c.

M & M' are the moments over the left and right hand supports at either end of the span l.

l, l_r, l_{r+1} &c., are the lengths of spans.

c_r, c_{r+1} &c., are the ordinates of the points of support at left hand of spans l_r, l_{r+1} &c., referred to the axis of abscissæ.

E represents the modulus of elasticity.

I represents the moment of inertia of the beam, referred to its neutral axis.

P represents a load, or element of load, within the span l.

$\overset{l}{\Sigma} P$ indicates the summation of all loads P between 0 and l.

$\underset{r}{\Sigma} P$, $\underset{r+1}{\Sigma} P$, &c., indicate the summation of all the loads P, in the spans l_r, l_{r+1} &c.

a is the distance of point of application in the span l of the load P, from the left hand support.

q_r, q_{r+1} &c., are the amounts of uniformly distributed load, per unit of length, in the spans l_r, l_{r+1} &c.

V_o & V_l are the vertical shearing forces at 0 and at l.

A & A' are the parts of the reactions at the left and right hand extremities of a span l, which are in equilibrium with the loads on that span.

$\overset{l}{\Sigma} P\,(l - a)$ means the summation of all the loads in the span l, each multiplied by the distance of its point of application from the right hand support; and

$\overset{l}{\Sigma} Pa$, the same sum multiplied by the distance of its point of application from the left hand support.

We pass now to the development of the general formulæ required for the calculation of pivot drawbridges.

§ 9. EQUATIONS FOR THE REACTIONS OF A GIRDER CONTINUOUS OVER THREE SUPPORTS.—Fig. 17. In **17**, above, make $r=2$, $r-1=1$, $r+1=3$,

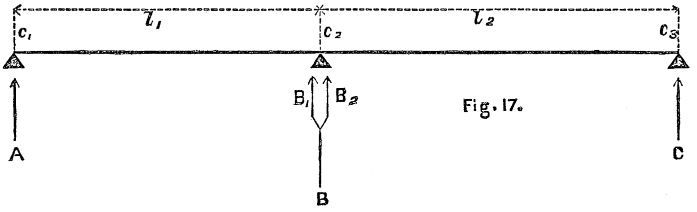


Fig. 17.

a in the span $l_1 = el_1$, and a in the span $l_2 = l_2 - gl_2$. M_1 and M_3 are each $=0$, the ends being free (not walled in). Hence **17** becomes

$$2\,M_2\,(l_1+l_2) = -\left[\frac{c_1-c_2}{l_1}+\frac{c_3-c_2}{l_2}\right] 6\,EI - \frac{1}{l_1}\,\Sigma\, Pe\,l_1\,(l_1^2 - e^2\,l_1^2)$$
$$-\frac{1}{l_2}\,\Sigma\, Qgl_2\,(l_2^2 - g^2\,l_2^2).$$

For convenience in writing, put the term $\left[\frac{c_1-c_2}{l_1}+\frac{c_3-c_2}{l_2}\right] 6\,EI = Y$, and find the value of M_2:

28. $$M_2 = \frac{-Y - \frac{1}{l_1}\,\Sigma\, Pe\,l_1\,(l_1^2 - e^2\,l_1^2) - \frac{1}{l_2}\,\Sigma\, Qgl_2\,(l_2^2 - g^2\,l_2^2)}{2\,(l_1+l_2)}.$$

Now, from **20** and **21**, introducing the new notation as just now in **17**, and making M_1 and $M_3=0$:

29. $$A = \frac{1}{l_1}\Big[M_2 + \Sigma\, P(l_1 - el_1)\Big].$$

30. $$B = B_1 + B_2 = \frac{1}{l_1}\Big[-M_2 + \Sigma\, Pel_1\Big] + \frac{1}{l_2}\Big[-M_2 + \Sigma\, Qgl_2\Big].$$

31. $$C = \frac{1}{l_2}\Big[M_2 + \Sigma\, Q\,(l_2 - gl_2)\Big].$$

Substituting the value of M_2 found above:

32. $$A = \frac{-Y - \frac{1}{l_1}\,\Sigma\, Pel_1\,(l_1^2 - e^2\,l_1^2) - \frac{1}{l_2}\,\Sigma\, Qgl_2\,(l_2^2 - g^2\,l_2^2)}{2\,l_1\,(l_1+l_2)}$$
$$+\frac{\Sigma\, P\,(l_1 - el_1)}{l_1}.$$

33. $$B = \frac{Y + \frac{1}{l_1}\,\Sigma\, Pel_1\,(l_1^2 - e^2\,l_2^2) + \frac{1}{l_2}\,\Sigma\, Qgl_2\,(l_2^2 - g^2\,l_2^2)}{2\,l_1\,l_2}$$
$$+\frac{\Sigma\, P\,(l_1 - el_1)}{l_1} + \frac{\Sigma\, Q\,(l_2 - gl_2)}{l_2}.$$

34. $$C = \frac{-Y - \frac{1}{l_1}\Sigma\, P e l_1 (l_1^2 - e^2 l_1^2) - \frac{1}{l_2}\Sigma\, Q g l_2 (l_2^2 - g^2 l_2^2)}{2\, l_1 (l_1 + l_2)} + \frac{\Sigma\, Q\,(l_2 - g l_2)}{l_2}.$$

Generally $l_1 = l_2 = l$, and the points of support are on a level, in which last event Y vanishes. Then after a few changes we get:

35. $A = \Sigma\left[\frac{P}{4}(4 - e\,[5 - e^2])\right] - \Sigma\left[\frac{Q}{4}\,g\,(1 - g^2)\right]$,

36. $B = \Sigma\left[\frac{P}{2}\,e\,(3 - e^2)\right] + \Sigma\left[\frac{Q}{2}\,g\,(3 - g^2)\right]$,

37. $C = \Sigma\left[\frac{Q}{4}(4 - g\,[5 - g^2])\right] - \Sigma\left[\frac{P}{4}\,e\,(1 - e^2)\right]$,

which are convenient forms for calculation. If further $e = g$ and $P = Q$ [equal spans, supports on a level, equal panel lengths and corresponding panel points equally loaded]:

38. $A = C = \Sigma\, P\left[1 - \frac{e\,(3 - e^2)}{2}\right]$.

39. $B = \Sigma\, P\,[e\,(3 - e^2)]$.

For uniformly distributed loads, q_1 in span l_1 and q_2 in span l_2, for $Y = 0$, *i.e.* the three supports on a level, $r - 1 = 1$, $r = 2$, and $r + 1 = 3$, $M_1 = M_3 = 0$ as before, **17***a* would become:

$$2\,M_2\,(l_1 + l_2) = -\tfrac{1}{4}\,q_1\,l_1^3 - \tfrac{1}{4}\,q_2\,l_2^3.$$

Similarly we should have for values of A, B, and C, derived from **20** and **21**, substituting

$$\int_0^{l_1} q_1\,dx\,(l_1 - x) \text{ for } \Sigma\, P\,(l - a) \text{ in span } l_1,$$

and $\int_0^{l_2} q_2\,dx.\,x$ for the same in span l_2,

also $\int_0^{l_1} q_1\,dx.\,x$ for $\Sigma\, Pa$ in span l_1,

and $\int_0^{l_2} q_2\,dx\,(l_2 - x)$ for the same in span l_2:

40. $A = \frac{1}{l_1}\left[M_2 + \tfrac{1}{2}\,q_1\,l_1^2\right]$.

41. $B=\frac{1}{l_1}\left[-M_2+\frac{1}{2}q_1 l_1^2\right]+\frac{1}{l_2}\left[-M_2+\frac{1}{2}q_2 l_2^2\right].$

42. $C=\frac{1}{l_2}\left[M_2+\frac{1}{2}q_2 l_2^2\right].$

Or substituting value of M_2:

43. $A=\frac{-\frac{1}{4}(q_1 l_1^3+q_2 l_2^3)}{2(l_1+l_2)\,l_1}+\frac{1}{2}q_1 l_1.$

44. $B=\frac{\frac{1}{4}(q_1 l_1^3+q_2 l_2^3)}{2\,l_1 l_2}+\frac{1}{2}(q_1 l_1+q_2 l_2).$

45. $C=\frac{-\frac{1}{4}(q_1 l_1^3+q_2 l_2^3)}{2(l_1+l_2)\,l_2}+\frac{1}{2}q_2 l_2.$

These are sometimes found in the forms

43*a*. $A=\frac{3\,q_1 l_1^3+4q_1 l_1^2 l_2-q_2 l_2^3}{8l_1(l_1+l_2)}.$

44*a*. $B=\frac{q_1 l_1^4+5\,q_1 l_1^3 l_2+4\,q_1 l_1^2 l_2^2+4\,q_2 l_1^2 l_2^2+5\,q_2 l_1 l_2^3+q_2 l_2^4}{8\,l_1 l_2(l_1+l_2)}.$

45*a*. $C=\frac{3\,q_2 l_2^3+4\,q_2 l_1 l_2^2-q_1 l_1^3}{8\,l_2(l_1+l_2)}.$

If in these $l_1=l_2=l$,

46. $A=\frac{l}{16}(7\,q_1-q_2).$

47. $B=\frac{10l}{16}(q_1+q_2).$

48. $C=\frac{l}{16}(7\,q_2-q_1).$

Finally for $q_1=q_2$ (equal spans, supports on a level, both spans equally an uniformly loaded):

49. $A=C=\frac{3}{8}\,ql.$

50. $B=\frac{10}{8}\,ql.$

§ 10. Equations for the Reactions of a Girder continuous ove four Supports. — Fig. 18. Making in **17**, r first $=2$ and then $=3$, first M and again M_4 become $=0$, and the equation becomes

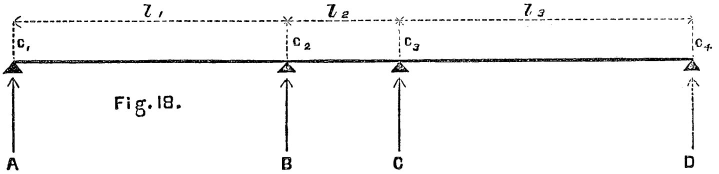


Fig. 18.

$$2\,M_2\,(l_1+l_2)+M_3\,l_2=-Y_1-\frac{1}{l_1}\Sigma\,Px(l_1^2-x_2)-\frac{1}{l_2}\Sigma\,Qy\,(l_2-y)\,(2\,l_2-y),\ \text{and}$$

$$M_2\,l_2+2\,M_3\,(l_2+l_3)=-Y_2-\frac{1}{l_2}\Sigma\,Qy\,(l_2^2-y^2)-\frac{1}{l_3}\Sigma\,Rz\,(l_3-z)\,(2\,l_3-z),$$

designating for brevity $\left[\frac{c_1-c_2}{l_1}+\frac{c_3-c_2}{l_2}\right]\,6\,EI$ by Y_1 and $\left[\frac{c_2-c_3}{l_2}+\frac{c_4-c_3}{l_3}\right]\,6\,EI$ by Y_2, replacing also the general load P and distance a by P, Q, R and x, y, z in the spans l_1, l_2, l_3, respectively. Solving these equations for M_2 and M_3, we obtain :

51. $$M_2=\frac{\left[Y_1+\frac{1}{l_1}\Sigma\,Px\,(l_1^2+x^2)+\frac{1}{l_2}\Sigma\,Qy(l_2-y)\,(2\,l_2-y)\right]2\,(l_2+l_3)}{l_2^2-4\,(l_2+l_3)\,(l_1+l_2)}$$
$$-\frac{\left[Y_2+\frac{1}{l_2}\Sigma\,Qy\,(l_2^2-y^2)+\frac{1}{l_3}\Sigma\,Rz\,(l_3-z)\,(2\,l_3-z)\right]l_2}{l_2^2-4\,(l_2+l_3)\,(l_1+l_2)}.$$

52. $$M_3=\frac{\left[Y_2+\frac{1}{l_2}\Sigma\,Qy\,(l_2^2-y^2)+\frac{1}{l_3}\Sigma\,Rz\,(l_3-z)\,(2\,l_3-z)\right]2\,(l_1+l_2)}{l_2^2-4\,(l_2+l_3)\,(l_1+l_2)}$$
$$-\frac{\left[Y_1+\frac{1}{l_1}\Sigma\,Px\,(l_1^2-y^2)+\frac{1}{l_2}\Sigma\,Qy\,(l_2-y)\,(2\,l_2-y)\right]l_2}{l_2^2-4\,(l_2+l_3)\,(l_1+l_2)}.$$

Comparing these, we find that the first quantity enclosed in [] in the value of M_2 is entirely similar with the second quantity enclosed in [], in the value of M_3, and that the same similarity occurs between the second term of M_2 and the first of M_3, so that for convenience we may write,

53. $$M_2=\frac{2\,(l_2+l_3)\,V-l_2\,W}{U}.$$

54. $$M_3=\frac{2\,(l_1+l_2)\,W-l_2\,V}{U}.$$

From **20** and **21**, making $M_1=M_4=0$, we obtain

55. $$A=\frac{1}{l_1}\Big[M_2+\Sigma\,P\,(l_1-x)\Big].$$

56. $$B=\frac{1}{l_1}\Big[-M_2+\Sigma\,Px\Big]+\frac{1}{l_2}\Big[-M_2+M_3+\Sigma\,Q\,(l_2-y)\Big].$$

57. $$C=\frac{1}{l_2}\Big[M_2-M_3+\Sigma\,Qy\Big]+\frac{1}{l_3}\Big[-M_3+\Sigma\,R\,(l_3-z)\Big].$$

58. $$D=\frac{1}{l_3}\Big[M_3+\Sigma\,Rz\Big].$$

Substituting the values of M_2 and M_3 given above, and reducing, these become:

59. $$A = \frac{2(l_2 + l_3)V - l_2 W}{l_1 U} + \frac{\Sigma P(l_1 - x)}{l_1}.$$

60. $$B = \frac{(2l_1 + l_2)(l_1 + l_2)W - [2(l_1 + l_2)(l_2 + l_3) + l_1 l_2]V}{l_1 l_2 U}$$
$$+ \frac{\Sigma Px}{l_1} + \frac{\Sigma Q(l_2 - y)}{l_2}.$$

61. $$C = \frac{(2l_3 + l_2)(l_2 + l_3)V - [2(l_1 + l_2)(l_2 + l_3) + l_2 l_3]W}{l_2 l_3 U}$$
$$+ \frac{\Sigma Qy}{l_2} + \frac{\Sigma R(l_3 - z)}{l_3}.$$

62. $$D = \frac{2(l_1 + l_2)W - l_2 V}{l_3 U} + \frac{\Sigma Rz}{l_3}.$$

Usually Y_1 and Y_2 will $= 0$, the supports being on the same level, $l_1 = l_3 = l$, $l_2 = ml$, $x = el$, $l_3 - z = gl$, and $Q = 0$. Under these conditions

$$V = \Sigma Pel^2(1 - e^2),$$
$$W = \Sigma Rgl^2(1 - g^2),$$
$$U = l^2[m^2 - 4(1 + m^2)], \text{ and}$$

63. $$A = \Sigma P(1 - e)\left[1 - \frac{2e(1 + m)(1 + e)}{3m^2 + 8m + 4}\right]$$
$$+ \Sigma R(1 - g)\frac{mg(1 + g)}{3m^2 + 8m + 4}.$$

64. $$B = \Sigma Pe\left[1 + \frac{(2 + 5m + 2m^2)(1 - e^2)}{m(3m^2 + 8m + 4)}\right]$$
$$- \Sigma Rg\frac{(2 + 3m + m^2)(1 - g^2)}{m(3m^2 + 8m + 4)}.$$

65. $$C = \Sigma Rg\left[1 + \frac{(2 + 5m + 2m^2)(1 - g^2)}{m(3m^2 + 8m + 4)}\right]$$
$$- \Sigma Pe\frac{(2 + 3m + m^2)(1 - e^2)}{m(3m^2 + 8m + 4)}.$$

66. $$D = \Sigma R(1 - g)\left[1 - \frac{2g(1 + m)(1 + g)}{3m^2 + 8m + 4}\right]$$
$$+ \Sigma P(1 - e)\frac{me(1 + e)}{3m^2 + 8m + 4}.$$

If further the loading be symmetrical, that is, if $e = g$ and $P = R$, a further reduction is possible:

67. $$A = D = \Sigma P (1 - e) \left[1 - \frac{e (m + 2) (1 + e)}{3 m^2 + 8 m + 4}\right].$$

68. $$B = C = \Sigma P e \left[1 + \frac{(m + 2) (1 - e^2)}{3 m^2 + 8 m + 4}\right].$$

Similarly for uniformly distributed loading, ΣP will become $\int_0^{l_1} q_1 \, dx$, $\Sigma Q, \int_0^{l_2} q_2 \, dx$, $\Sigma R, \int_0^{l_3} q_3 \, dx$, and we shall have from formulæ **59 - 62**:

69. $$A = \frac{2 (l_2 + l_3) [\frac{1}{4} q_1 l_1^3 + \frac{1}{4} q_2 l_2^3] - l_2 [\frac{1}{4} q_2 l_2^3 + \frac{1}{4} q_3 l_3^3]}{l_1 [l_2^2 - 4 (l_1 + l_2) (l_2 + l_3)} + \frac{1}{2} q_1 l_1.$$

70. $$B = \frac{(l_1 + l_2) (2 l_1 + l_2) [\frac{1}{4} q_2 l_2^3 + \frac{1}{4} q_3 l_3^3]}{l_1 l_2 (l_2^2 - 4 (l_1 + l_2) (l_2 + l_3)]}$$
$$- \frac{[l_1 l_2 + 2 (l_1 + l_2) (l_2 + l_3)] (\frac{1}{4} q_1 l_1^3 + \frac{1}{4} q_2 l_2^3)}{l_1 l_2 [l_2^2 - 4 (l_1 + l_2) (l_2 + l_3)]}.$$

71. $$C = \frac{(l_3 + l_2) (2 l_3 + l_2) [\frac{1}{4} q_1 l_1^3 + \frac{1}{4} q_2 l_2^3]}{l_1 l_2 [l_2^2 - 4 (l_1 + l_2) (l_2 + l_3)]}$$
$$- \frac{[l_1 l_2 + 2 (l_1 + l_2) (l_2 + l_3)] (\frac{1}{4} q_2 l_2^3 + \frac{1}{4} q_3 l_3^3)}{l_1 l_2 [l_2^2 - 4 (l_1 + l_2) (l_2 + l_3)]}.$$

72. $$D = \frac{2 (l_1 + l_2) [\frac{1}{4} q_2 l_2^3 + \frac{1}{4} q_3 l_3^3] - l_2 [\frac{1}{4} q_1 l_1^3 + \frac{1}{4} q_2 l_2^3]}{l_3 [l_2^2 - 4 (l_1 + l_2) (l_2 + l_3)]} + \frac{1}{2} q_3 l_3.$$

When again $l_1 = l_3 = l$, $l_2 = ml$ (m for drawbridges would be a proper fraction), also $q_1 = q_3 = q$:

73. $$A = D = \frac{1}{2} q l \left[1 - \frac{2 + m}{2 (3 m^2 + 8 m + 4)}\right] - \frac{1}{2} q_2 l_2 \frac{m^3 (2 + m)}{2 (3 m^2 + 8 m + 4)}.$$

74. $$B = C = \frac{1}{2} q l \left[1 + \frac{2 + m}{2 (3 m^2 + 8 m + 4)}\right]$$
$$+ \frac{1}{2} q_2 l m \left[1 + \frac{m^2 (2 + m)}{2 (3 m^2 + 8 m + 4)}\right].$$

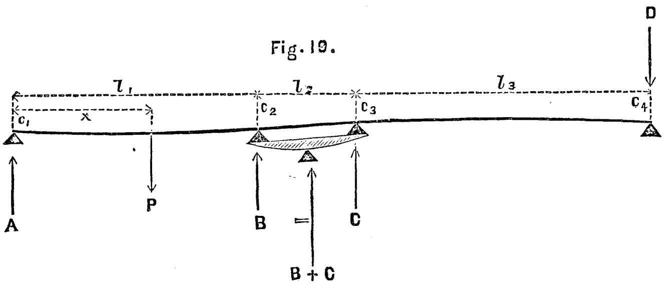
Finally when $q_2 = q$ and $m = 1$, that is $l_1 = l_2 = l_3 = l$,

75. $A = D = \frac{4}{10} q l$,

76. $B = C = \frac{11}{10} q l$.

There remain to be developed the equations for the very peculiar cases, when two of the supports are themselves supported on the ends of an inflexible body, which, in turn, rests upon a knife edge or pivot at its centre. (Fig. 19.) It is evident that, for equilibrium, the reactions of such supported supports must be equal. An incidental quality is that, when such supports are *not* symmetrically loaded, the ordinate of one support, measured from the line connecting them when in a horizontal or symmetrically loaded position, equals the ordinate of the other measured in the opposite direction, or, for a three span girder, $c_2 = -c_3$.* The calculation of the equations for the reactions under these conditions is tedious but not difficult, after the method to be followed and the circumstances to be taken into account are once clearly imagined.

Fig. 19.



§ 11. EQUATIONS FOR THE REACTIONS OF A GIRDER CONTINUOUS OVER FOUR SUPPORTS, THE TWO CENTRE ONES RESTING UPON AN INFLEXIBLE BODY, WHICH IS PIVOTED AT THE CENTRE. — For our cases of unsymmetrical loading, we must make $Q = R = 0$. We then have the main condition of equilibrium, that $B = C$ (see fig. and equations in § 10).

77. $$B = \frac{1}{l_1}\Big[-M_2 + \Sigma Px\Big] + \frac{1}{l_2}\Big[M_3 - M_2\Big] = \frac{1}{l_2}\Big[M_2 - M_3\Big] - \frac{M_3}{l_3} = C,$$

or $$-\frac{M_2}{l_1} - \frac{2\,M_2}{l_2} + \frac{2\,M_3}{l_2} + \frac{M_3}{l_3} = -\frac{\Sigma\,Px}{l_1} = T;$$

also,

78. $$2\,M_2\,(l_1 + l_2) + M_3\,l_2 = -Y_1 - \frac{1}{l_1}\,\Sigma\,Px\,(l_1^2 - x^2) = R.$$

79. $$M_2\,l_2 + 2\,M_3\,(l_2 + l_3) = -Y_2 = S.$$

80. From **79**, $M_2 = \dfrac{S - 2\,M_3\,(l_2 + l_3)}{l_2}$. Substituting in **78** and finding value of M_3, —

---

* This is exactly true when the pivot is situated in the line joining B and C, and equi-distant from these points. There is no difficulty about developing the equations when $c_2 = -nc_3$, or even $f(c_2) = -f(c_3)$.

81. $M_3 = \frac{l_2 R - 2(l_1 + l_2) S}{l_2^2 - 4(l_1 + l_2)(l_2 + l_3)}$. Substituting **80** in **77** will give

82. $M_3 = \frac{l_1 l_2^2 l_3 T + (l_2 l_3 + 2 l_1 l_3) S}{2 l_2^2 l_3 + 2 l_2 l_3^2 + 4 l_1 l_2 l_3 + 4 l_1 l_3^2 + l_1 l_2^2}$.

Make now $l_1 = l_3 = l$, and $l_2 = ml$, and put **81** = **82**, we get,—

83. $S - R = ml^2 T$. Substituting values of S, R, and T, we get, finally,

84. $Y_2 - Y_1 = \frac{1}{l} \Sigma Px (l^2 - x^2 + ml^2)$.

But $-Y_1 = -\left[\frac{c_1 - c_2}{l_1} + \frac{c_3 - c_2}{l_2}\right] 6 EI$,

and $-Y_2 = -\left[\frac{c_2 - c_3}{l_2} + \frac{c_4 - c_3}{l_3}\right] 6 EI$, also $c_1 = c_4 = 0$, and $c_2 = -c_3$;

$\therefore Y_1 = -\left[\frac{2c + mc}{ml}\right] 6 EI$, and $Y_2 = \left[\frac{2c + mc}{ml}\right] 6 EI$, and

85. $-Y_1 = \frac{1}{2l} \Sigma Px (l^2 - x^2 + ml^2)$

86. $-Y_2 = -\frac{1}{2l} \Sigma Px (l^2 - x^2 + ml^2)$.*

Substituting values of $-Y_1$ and $-Y_2$ in **73** and **79**, and reducing, we get,—

87. $M_2 = \frac{-\left(\frac{2+m}{2l^2}\right) \Sigma Px (l^2 - x^2) + \frac{(2 + 3m) m}{2} \Sigma Px}{3m^2 + 8m + 4}$.

88. $M_3 = \frac{-\left(\frac{2+m}{2l^2}\right) \Sigma Px (l^2 - x^2) - \frac{(2 + 3m) m}{2} \Sigma Px}{3m^2 + 8m + 4}$.

These values substituted in the general equations for A, B, C, and D (**55–58**) $[Q = R = 0,\ l_1 = l_3 = l,\ l_2 = ml]$ give finally,—

89. $A = \frac{-(2+m) \Sigma Px (l^2 - x^2) + (2 + 3m) ml^2 \Sigma Px}{2 l^3 (3m^2 + 8m + 4)} + \frac{1}{l} \Sigma P(l - x)$.

90. $B = C = \frac{(2 + m) \Sigma Px (l^2 - x^2)}{2 l^3 (3m^2 + 8m + 4)} + \frac{\Sigma Px}{2l}$.

91. $D = -\frac{(2 + m) \Sigma Px (l^2 - x^2) + (2 + 3m) ml^2 \Sigma Px}{2 l^3 (3m^2 + 8m + 4)}$,

and $A + B + C + D = \Sigma P$, as they should.

To bring these equations to the same known quantities as equations **63–66**, we have but to substitute el for x.

---

* From the values of Y_1 and Y_2, in terms of c, m, and l, and the same values in terms of ΣPx, m, and l, the value of c can readily be found, but it will contain the term EI.

§ 12. EQUATIONS FOR THE REACTIONS OF A GIRDER CONTINUOUS OVER THREE SUPPORTS, THE TWO RIGHT HAND ONES RESTING UPON AN INFLEXIBLE BODY, WHICH IS PIVOTED AT THE CENTRE. — Taking again for our case of unsymmetrical loading $Q=0$, $l_1=l$, $l_2=ml$, we have for the main condition of equilibrium $B=C$, or

$$\frac{1}{l}\Big[-M_2+\Sigma Px\Big]+\frac{1}{ml}\Big[-M_2\Big]-\frac{1}{ml}\Big[M_2\Big]=0,\text{ or}$$

92. $M_2=\frac{m\,\Sigma\,Px}{m+2}$. Also,

93. $2\,M_2\,(l+ml)=-Y-\frac{1}{l}\,\Sigma\,Px\,(l^2-x^2)$, or

94. $M_2=\dfrac{-Y-\frac{1}{l}\,\Sigma\,Px\,(l^2-x^2)}{2\,l\,(1+m)}$. From **92** and **93**,

95. $-Y=\dfrac{2\,l\,(1+m)\,m\,\Sigma\,Px+\frac{m+2}{l}\,\Sigma\,Px\,(l^2-x^2)}{m+2}$.

Substituting in **93**,

96. $M_2=\frac{m}{m+2}\,\Sigma\,Px$,* and substituting this in the general equations for A, B, and C (**29**-**31**).

97. $A=\frac{m}{l\,(m+2)}\,\Sigma\,Px+\frac{\Sigma\,P}{l}\,(l-x)$.

98. $B=C=\frac{\Sigma\,Px}{l\,(m+2)}$,

and $A+B+C$ is, as it should be, equal to $\Sigma\,P$.

Again, substituting el for x, we obtain A, B, and C, in terms of the same known quantities as in equations **35**-**37**.

§ 13. IN A GIRDER CONTINUOUS OVER FOUR LEVEL SUPPORTS, THE TWO CENTRE ONES PIVOTED (SAME AS IN § 11), TO FIND WHAT LOADING UNIFORMLY DISTRIBUTED, ADDED TO A GIVEN LOADING UNIFORMLY DISTRIBUTED OVER THE SPAN AB, WILL MAKE $D=0$. — We had for the unsymmetrical load (**91**), —

$$D=-\frac{(2+m)\,\Sigma\,Px\,(l^2-x^2)+(2+3\,m)\,ml^2\,\Sigma\,Px}{2\,l^3\,(3\,m^2+8\,m+4)}.$$

---

* Being the same as found (**92**), which shows that the principles of the lever alone suffice in this instance, and that these principles are but a special case of those of the elastic line.

For uniform loading over the whole span AB, we must substitute $\int_0^l q_1\,dx$ for ΣP, which gives,—

99. $$D = -\frac{\frac{(2+m)}{4}\,q_1 l^4 + (2+3m)\,mq_1\frac{l^4}{2}}{2\,l^3\,(3\,m^2+8\,m+4)} = \text{unsymmetrical } D.$$

From **70**, with $q_2 = q$, we have,—

100. $$D = \tfrac{1}{2}\,ql\left[1 - \frac{(2+m)}{2\,(3m^2+8m+4)}\right] - \tfrac{1}{2}\,ql\,\frac{m^3\,(2+m)}{2\,(3m^2+8\,m+4)} = \text{syml. } D.$$

The condition is that unsymmetrical $D +$ symmetrical D shall equal 0, or

$$-\frac{\frac{(2+m)}{4}\,q_1 l^4 + (2+3\,m)\,mq_1\frac{l^4}{2}}{2\,l^3\,(3\,m^2+8\,m+4)} = \tfrac{1}{2}\,ql\left[1 - \frac{(2+m)}{2\,(3\,m^2+8\,m+4)}\right] - \tfrac{1}{2}\,ql\,\frac{m^3\,(2+m)}{2\,(3\,m^2+8\,m+4)},$$

from which is found,

101. $$q = \frac{q_1\,(2+5\,m+6m^2)}{2\,(6+15\,m+6\,m^2-2\,m^3-m^4)} = \text{uniform load per unit of}$$

length over all three spans, necessary to make $D = 0$, when span AB is loaded with q_1 per unit of length in addition to q, and the supports are in the condition above stated.

§ 14. In a Girder continuous over Two Unequal Spans, to find what Loading uniformly distributed, added to a given Load uniformly distributed over the Span AB, will make $C = 0$.—We had (**45**a),—

$$C = \frac{3\,q_2\,l_2^3 + 4\,q_2\,l_1\,l_2^2 - q_1\,l_1^3}{8\,l_2\,(l_1 + l_2)}.$$

For $q_2 = 0$, or unsymmetrical loading, and $l_2 = ml$, we have

102. $$\text{Unsymmetrical } C = -\frac{q_1\,l}{8\,m\,(1+m)}.$$

For $q_1 = q_2 = q$, or symmetrical loading,

103. $$\text{Symmetrical } C = \frac{3\,ql\,m^3 + 4\,ql\,m^2 - ql}{8\,m\,(1+m)},$$ or putting

$$-\frac{q_1\,l}{8\,m\,(1+m)} + \frac{3\,ql\,m^3 + 4\,ql\,m^2 - ql}{8\,m\,(1+m)} = 0,$$ we obtain,

104. $$q = \frac{q_1}{3\,m^3 + 4\,m^2 - 1}.$$

PART III.

§ 15. NUMERICAL EXAMPLE. (See Plate I.) — *Loading.* The assumed dead load of each truss (which is in 3 spans of 90, 20, and 90 feet, and made continuous over them all) is taken at 1000 lbs., the snow load at 200 lbs., and the live load at 1400 lbs. per lineal foot. Each panel is 18 feet long, except the centre one, which is 20 feet long. The above loads, concentrated at the panel points and points of support, are as follows, in tons: —

Over each end support	4.5 tons dead load. 0.9 tons snow ,, 6.3 tons live ,,
At each panel point	9.0 tons dead load. 1.8 tons snow ,, 12.6 tons live ,,
Over each centre support . . .	9.5 tons dead load. 1.9 tons snow ,, 13.3 tons live ,,

LEVER ARMS* FOR THE FIRST SYSTEM (IN FEET).

$U_{1 \& 2}$	19.929	L_1	18.5	S_1	172.01	T_2	184.02
$U_{3 \& 4}$	22.92	$L_{2 \& 3}$	21.5	S_3	211.62	T_4	222.43
U_5	25.909	$L_{4 \& 5}$	24.5	S_5	251.44	P_2	∞
U_6	26.	L_6	26.	P_1	222.	T_6	$\infty \sin \phi$

LEVER ARMS* FOR THE SECOND SYSTEM (IN FEET).

$U_{2 \& 3}$	21.425	$L_{1 \& 2}$	20.	S_2	191.77	T_1	165.01
$U_{4 \& 5}$	24.414	$L_{3 \& 4}$	23.	S_4	231.52	T_3	203.17
U_6	26.	L_5 & L_6	26.			T_5	241.72

Formulæ for the Reactions. — For our case $m = \frac{2}{9}$. Substituting this in the equations for reactions, and making $e = g = k$, they reduce to the following: —

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* See Plate II.

Formulæ for four level supports; spans *AB* and *CD* symmetrically loaded, from **67** and **68**,

$$\left\{\begin{aligned} A = D &= \frac{\Sigma P}{8}(8 - 11k + 3k^3). \\ B = C &= \frac{\Sigma P}{8}(11k - 3k^3). \end{aligned}\right.$$

Formulæ for four level supports; span *AB* only loaded, from **63–66**,

$$\left\{\begin{aligned} A &= \frac{\Sigma P}{80}(80 - 113k + 33k^3). \\ B &= \frac{\Sigma P}{16}(55k - 39k^3). \\ C &= -\frac{33}{16}\Sigma P(k - k^3). \\ D &= \frac{3}{80}\Sigma P(k - k^3). \end{aligned}\right.$$

Formulæ for four supports, when *B* and *C* are "pivoted" and *AB* only loaded, from **89–91**,

$$\left\{\begin{aligned} A &= \frac{\Sigma P}{80}(80 - 91k + 15k^3). \\ B = C &= \frac{\Sigma P}{16}(11k - 3k^3). \\ D &= -\frac{\Sigma P}{80}(19k - 15k^3). \end{aligned}\right.$$

Formulæ for three supports, when *B* and *C* are "pivoted" and *AB* only loaded, from **97–98**,

$$\left\{\begin{aligned} A &= \frac{\Sigma P}{10}(10 - 9k). \\ B = C &= \frac{9}{20}\Sigma Pk. \end{aligned}\right.$$

In all cases $A + B + C + D = \Sigma P$, except the first written, when $A + B + C + D = 2\Sigma P$, as it should.

Example. — Snow load in the first system: —

$P = 1.8$ & $k = .2$; $P = 1.8$ & $k = .6$; $P = 1.9$ & $k = 1$

∴ by first formula, $A = D = 1.8 \times .984 = 1.7712$

$B = C = 1.8 \times 1.016 + 1.9 = 3.7288$

Snow load in the second system: —

$P = 0.9$ & $k = 0$; $P = 1.8$ & $k = .4$; $P = 1.8$ & $k = .8$

∴ by same formula as above:

$A = D = 1.8 \times .566 + 0.9 = 1.9188$

$B = C = 1.8 \times 1.434 = 2.5812$

The reactions of the girder, for the several cases given in Part I., and found in a manner similar to the above, are given in the following Table: —

REACTIONS (IN TONS).

Manner of Support.	Load.	FIRST SYSTEM.				SECOND SYSTEM.			
		A	*B*	*C*	*D*	*A*	*B*	*C*	*D*
Case I.	Dead,	0.	27.5	27.5	0.	0.	22.5	22.5	0.
	Snow,	0.	5.5	5.5	0.	0.	4.5	4.5	0.
		0.	33.	33.	0.	0.	27.	27.	0.
Case II.	Dead,	0.	27.5	27.5	0.	0.	22.5	22.5	0.
	Snow,	1.7712	3.7288	3.7288	1.7712	1.9188	2.5812	2.5812	1.9188
	Live,	12.3984	26.1016	26.1016	12.3984	13.4316	18.0684	18.0684	13.4316
		14.1696	57.3304	57.3304	14.1696	15.3504	43.1496	43.1496	15.3504
Case III.(*a*)	Dead,	0.	27.5	27.5	0.	0.	22.5	22.5	0.
	Snow,	1.77120	3.7288	3.7288	1.77120	1.91880	2.5812	2.5812	1.91880
	Live,	12.12624	41.0704	—14.9688	0.27216	13.13676	34.2846	—16.2162	0.29484
		13.89744	72.2992	16.2600	2.04336	15.05556	59.3658	8.8650	2.21364
Case III. (*b*)	Dead,	0.	27.5	27.5	0.	0.	22.5	22.5	0.
	Snow,	0.	5.5	5.5	0.	0.	4.5	4.5	0.
	Live,	17.458	10.521	10.521	0.	17.892	6.804	6.804	0.
		17.458	43.521	43.521	0.	17.892	33.804	33.804	0.
Case IV. (*a*)		Same reactions	as for *III.* (*a*)	[because $D=+$]					
Case IV. (*b*)	Dead,	0.	27.5	27.5	0.	0.	22.5	22.5	0.
	Snow,	1.7712	3.7288	3.7288	1.7712	1.9188	2.5812	2.5812	1.9188
	Live,	14.9282	13.0508	13.0508	—2.5298	15.6618	9.0342	9.0342	—2.2302
		16.6994	44.2796	44.2796	—0.7586	17.5806	34.1154	34.1154	—0.3114
Case V. (*a*)	$\frac{2}{3}$ Dead,	0.	18.33333+	18.33333+	0.	0.	15.	15.	0.
	$\frac{1}{3}$ Dead,	2.95200	6.21466+	6.21466+	2.95200	3.19800	4.3020	4.3020	3.19800
	Snow,	1.77120	3.72880	3.72880	1.77120	1.91880	2.5812	2.5812	1.91880
	Live,	12.12624	41.07040	—14.96880	0.27216	13.13676	34.2846	—16.2162	0.29484
		16.84944	69.3472	13.308	4.99536	18.25356	56.1678	5.667	5.41164
Case V. (*b*)	$\frac{2}{3}$ Dead,	0.	18.33333+	18.33333+	0.	0.	15.	15	0.
	$\frac{1}{3}$ Dead,	2.9520	6.21466+	6.21466+	2.9520	3.1980	4.3020	4.3020	3.1980
	Snow,	1.7712	3.72880	3.72880	1.7712	1.9188	2.5812	2.5812	1.9188
	Live,	14.9282	13.05080	13.05080	—2.5298	15.6618	9.0342	9.0342	—2.2302
		19.6514	41.3276	41.3276	2.1934	20.7786	30.9174	30.9174	2.8866

Strain Sheets.—(See Plates III.-IX.) The figures represent resultant strains in tons of 2,000 lbs. (+) means tension, (—) means compression. To abbreviate calculation, the dead load has been taken as though it were concentrated at the lower panel points; it is more exact, and in principle just as easy to follow out, to take the own weight as divided into two main parts, (1) that of the truss, and (2) that of the roadway platform, and each as though applied at that part of the skeleton outline of the bridge at which it *is* applied. A close approximation is to take the platform weight + $\frac{1}{2}$ weight of truss, at lower panel points, and $\frac{1}{2}$ weight of truss at upper panel points in "through" bridges; reverse this arrangement in "deck" bridges, and other combinations will readily suggest themselves in their proper places.

To illustrate the manner in which these Strain Sheets have been calculated, the computation for *Case II.* is next given, worked throughout *strictly* according to Ritter's method, although for chord members another way may sometimes be shorter, as alluded to in Part I.

Equations for finding the Strains in the several members of the Draw, loaded and supported (that is, reacted on) as in Case II.—By making the proper substitutions for loads and reactions, all the other cases can be calculated according to the same formulæ.* The centre panel diagonals are made to take tension only; when the resultant strain on one of them comes out *minus* therefore, neglect that diagonal, and make the calculation over again [for that and for such other members as depend upon such change], supposing now only the other diagonal in place. All other members are made to take either the compression or extension that they may be called upon to withstand.

FIRST SYSTEM.

$$U_{1\,\&\,2} \times 19.929 + 14.17 \times 18 = 0.$$
$$U_{3\,\&\,4} \times 22.92 + 14.17 \times (3 \times 18) - 23.4 \times (2 \times 18) = 0.$$
$$U_5 \times 25.909 + 14.17 \times (5 \times 18) - 23.4 \times [(2 + 4) \times 18] = 0.$$
$$U_6 \times 26. + 14.17 \times [(5 \times 18) + 20] + (57.33 - 24.7) \times 20 - 23.4 \times [((4 \times 18) + 20) + ((2 \times 18) + 20)] = 0.$$

$$-L_1 \times 18.5 + 14.17 \times 0 = 0.$$
$$-L_{2\,\&\,3} \times 21.5 + 14.17 \times (2 \times 18) - 23.4 \times 18 = 0.$$
$$-L_{4\,\&\,5} \times 24.5 + 14.17 \times (4 \times 18) - 23.4 \times [(1 + 3) \times 18] = 0.$$
$$-L_6 \times 26. + 14.17 \times (5 \times 18) - 23.4 \times [(2 + 4) \times 18] = 0.$$

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* The several equations have been written out in full, and without the slightest attempt at reduction; on the contrary, *all the elements of each term* have been conspicuously written and separated, so as to enable any one to follow the process of evolving the equations for each member in any case. In practice, no one, after doing a single day's work according to Ritter, would need to write out any equations as fully as done here, and for most members would not write them out at all, unless indeed "in his mind's eye."

$$S_1 \times 172.01 - 14.17 \times 222 = 0.$$
$$-T_2 \times 184.02 - 14.17 \times 222 + 23.4 \times (222 + 18) = 0.$$
$$S_3 \times 211.62 - 14.17 \times 222 + 23.4 \times (222 + 18) = 0.$$
$$-T_4 \times 222.43 - 14.17 \times 222 + 23.4 \times [(222 + 18) + (222 + (3 \times 18))] = 0.$$
$$S_5 \times 251.44 - 14.17 \times 222 + 23.4 \times [(222 + 18) + (222 + (3 \times 18))] = 0.$$

$$-P_1 = A.$$
$$-P_2 \times [222 + (5 \times 18)] - (57.33 - 24.7) \times [222 + (5 \times 18)] - 14.17 \times 222 + 23.4 \times [444 + (4 \times 18)] = 0.$$

$T_6 \sin\phi \, \infty - 14.17 \times \infty + 23.4 \times \infty + 23.4 \times \infty - (57.33 - 24.7) \times \infty = 0$, where ϕ is the angle T_6 makes with the horizontal.

$\therefore$ Sin $\phi = \frac{26}{32.803}$, and dividing by ∞:

$$T_6 \times \frac{26}{32.803} - 14.17 + 23.4 + 23.4 - 32.63 = 0.$$

SECOND SYSTEM.

$$U_{2\,\&\,3} \times 21.425 + (15.35 - 11.7) \times (2 \times 18) = 0.$$
$$U_{4\,\&\,5} \times 24.414 + 3.65 \times (4 \times 18) - 23.4 \times (2 \times 18) = 0.$$
$$U_6 \times 26 + 3.65 \times [(5 \times 18) + 20] + 43.15 \times 20 - 23.4\,[((3 \times 18) + 20) + (18 + 20)] = 0.$$

$$-L_{1\,\&\,2} \times 20 + (15.35 - 11.7) \times 18 = 0.$$
$$-L_{3\,\&\,4} \times 23 + 3.65 \times (3 \times 18) - 23.4 \times 18 = 0.$$
$$-L_5 \times 26 + 3.65 \times (5 \times 18) - 23.4\,[(3 \times 18) + 18] = 0.$$
$$-L_6 \times 26 + 3.65 \times [(5 \times 18) + 20] - 23.4\,[((3 \times 18) + 20) + (18 + 20)] + 43.15 \times 20 = 0.$$

$-T_1 \times 165.01 - 3.65 \times 222. = 0$ [or, revolving about second load point: $+T_1 \times \sqrt{36^2 - 26.907^2} + 3.65 \times 36 = 0$].

$$S_2 \times 191.77 - 3.65 \times 222 = 0.$$
$$-T_3 \times 203.17 - 3.65 \times 222 + 23.4 \times [222 + (2 \times 18)] = 0.$$
$$S_4 \times 231.52 - 3.65 \times 222 + 23.4 \times [222 + (2 \times 18)] = 0.$$
$$-T_5 \times 241.72 - 3.65 \times 222 + 23.4 \times [(222 + (2 \times 18)) + (222 + (4 \times 18))] = 0.$$

$$-P_2 \times [222 + (5 \times 18)] - 3.65 \times 222 + 23.4\,[444 + (6 \times 18)] - T_5 \times 241.72 - 43.15 \times [222 + (5 \times 18)] = 0.$$

$$T_6 \sin\phi - 3.65 + 23.4 + 23.4 - 43.15 = 0.$$

The other half girder must be made symmetrical, or, if desired (in the latched cases), can be calculated, either by continuing to make the supposed sections in the order of procedure from left to right, always introducing reactions as though they were merely minus loads (which *is* all they are), or by taking sections anew in the order of from right to left.

In this way have been found the strains in Plates III.–IX., and tabulated in Table (*A*), p. 50. Observe that the centre panel diagonals present a peculiar feature: the diagonal in service is a different one for the cases (*a*) and (*b*), and has a different value for the two, but each diagonal has the same value whether in *III.*, *IV.*, or *V.* $A + B$ is constant, in pivoted as well as fixed centre support draws:

In the first, $A + B <$ loads on AB.
In the second, $A + B >$ loads on AB.

A careful study of this table will prove instructive also as to choice of *kind* of supports for drawbridges. The table is not complete, however; there should be added the strains for the cases where there is no snow load, especially for *Case IV.* (*b*), and in the span *CD.* These may be obtained in two ways: (1) we could get the new reactions, by subtracting, for each support, the reactions produced there by the snow load from those already found, and then proceeding in the calculation of strains as before, or (2) by finding first the strains produced in each member by the snow load, and then algebraically subtracting these strains from those already found in the corresponding members. The snow strains are, however, of two kinds; once when the weight of the snow is all carried by the centre supports, and again when the truss carrying the snow loads rests on 4 level supports. The first mentioned snow strains obtain in *Cases I.* and *III.* (*b*), and must also be subtracted to get the strains without snow in *Cases V.* (*a*) and (*b*), whenever the end reactions without snow are to be the same they were with snow. (In *Cases V.* (*a*) and (*b*), it will be remembered, the end supports are lifted, just enough to prevent the end reactions from ever becoming less than a chosen plus quantity.) For, the end reactions remaining the same, the centre reactions only can have diminished, and must have diminished by just the weight of the snow; and the strains can have diminished algebraically, only by the strains produced by these two centre reactions, together equal to the snow load, acting as opposed to the several panel snow loads. Another view would be this: we had made *s*, the part of the own weight that was carried on the 4 supports, instead of on only the central ones, just large enough to securely keep the light end from rising in the event of maximum unsymmetrical loading. If now the snow melts off, it will render the light end insecure, and enough additional *s* must be taken out of the own weight to restore the original end reactions; that is, the new *s* must $=$ old $s +$ snow. The process is, the snow is removed from the 4 level supports, then a weight equal to it is taken off of the centre supports, and is distributed over the 4 level supports again, leaving, as a net result, the centre supports lighter, by the total snow load. Mechanically, this will necessitate a higher *position* of the end supports, at the end of the lift, in order that the same end reactions may be obtained without, as before were obtained with, the snow. Were, on the contrary, the end supports to remain in the same position, or at the same level, then, to get the strains in the trusses without the snow load, from those above with the snow on, we should have to subtract the snow strains produced by the snow load resting on 4 level supports.

Figs. 20 and 21, Plate X., show the two kinds of snow strains; Fig. 20, being those when the snow reactions are produced only at the centre supports, and are evidently simply $\frac{1}{6}$ the strains given for *Case I.* The others (Fig. 21) are the strains due a load of 200 lbs. per foot on a bridge skeleton without weight, and of the form under consideration, and supported on 4 level supports. Now subtract these strains in the several members, algebraically, from the corresponding strains in the table, remembering which set to use in the several cases, as above given. Fig. 20 has been used in *Case V.* (*b*) and Fig. 21 in *Case V.* (*a*), since $\frac{1}{3}$ dead was more than enough for s in this latter case, and $D = +$, even without the snow.

The resultant strains for all the cases will be found in Table (*B*), on p. 51.

While this book is going through the press, the author notices, in a work* recently issued, the following sentence: "But no book in common use gives us any method for determining the shearing stress under a partial load, a determination which is necessary before the bracing can be correctly proportioned." It is to be hoped, however, that no insuperable objection lies in the way to the attainment of such an object, and that such a book may be written. The foregoing has been an attempt in that direction; for the careful reader will have noticed that, by the method and the formulæ herein given, the effect of a load at any point or points, on a continuous girder, upon any of its members, may be calculated.

The method of strain calculation followed throughout has been what may be termed the "arithmetical" one; and the author has purposely presented the same purely and alone, and omitting, therefore, considerations of moment-curves, of vertical and horizontal shears, points of contrary flexure, and other paraphernalia which usually accompany treatises on continuous girders.

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* Graphical Method for the Analysis of Bridge Trusses, &c. Charles E. Greene. New York. 1875. Van Nostrand.

To calculate the absolute maxima and minima strains in the members of a continuous panel girder by the method that has been followed, the process would be like this : —

First find the strains in each member, under the own weight of the bridge.

Next find the strains in each member for the maximum panel live load on the first panel point, all the rest of the bridge being supposed to be without weight and unloaded.

Next place the panel live load on the second panel point, the rest of the bridge being supposed to be without weight and unloaded, and so on, for each panel point. By tabulating the strains thus found for each member of the bridge, a clear oversight will be gained over them. Then summing up all the minus strains liable to occur in any one member, with the strain due to the own weight, will give the maximum minus strain, and summing up its plus strains with the same will give the maximum plus strain that can come upon it by any possible combination of loads on the bridge.

TABLE (*A*) OF STRAINS.

Pieces.	*Case I.*	*Case II.*	*Case III.* (*a*)		*Case III.* (*b*)		*Case IV.* (*a*)		*Case IV.* (*b*)		*Case V.* (*a*)		*Case V.* (*b*)	
			Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*
U_1	0.	—12.8	—12.6	— 1.9	—15.8	0.	—12.6	— 1.9	—15.1	+ **0.7**	—15.0	— 4.5	—**17.8**	— 2.0
U_2	+ 9.1	—18.9	—18.2	+ 3.5	—26.2	+ 9.1	—18.2	+ 3.5	—25.0	+**10.3**	—26.0	— 4.5	—**33.0**	+ 2.1
U_3	+26.0	— 2.8	— 1.6	+17.5	—14.8	+26.0	— 1.6	+17.5	—12.5	+**28.3**	—14.0	+ 5.2	—**24.8**	+15.8
U_4	+48.8	+27.1	+28.6	+37.5	+11.9	+48.8	+28.6	+37.5	+14.6	+ **51.5**	+12.2	+20.3	— **1.8**	+35.1
U_5	+76.9	+72.1	+73.9	+63.2	+53.1	+76.9	+73.9	+63.2	+56.7	+**80.4**	+54.2	+42.7	+ 37.0	+60.7
U_6	+93.5	+**100.3**	+78.7		+80.1		+78.7		+83.8		+57.4		+62.6	
L_1	— 4.9	+ 3.3	+ 3.0	— 2.9	+ 5.6	— 4.9	+ 3.0	— 2.9	+ 5.3	— **5.1**	+ 5.9	0.	+ **8.2**	— 2.3
L_2	—13.9	+ 7.4	+ 6.7	— 8.5	+15.2	—13.9	+ 6.7	— 8.5	+13.7	—**15.5**	+14.5	— 0.7	+**21.5**	— 7.6
L_3	—30.2	— 5.6	— 6.8	—21.6	+ 5.9	—30.2	— 6.8	—21.6	+ 3.9	—**32.2**	+ 5.7	— 9.1	+**16.4**	—19.5
L_4	—52.9	— 36.9	—38.4	—41.7	—21.2	—52.9	—38.4	—41.7	—24.2	—**55.8**	—22.9	—25.5	— 8.0	—39.4
L_5	—80.3	— 79.3	—81.1	—66.7	—60.8	—80.3	—81.1	—66.7	—64.1	—**83.7**	—62.1	—46.9	— 44.4	—63.9
L_6	—93.5	—100.3	—**102.3**		—93.5		—**102.3**		—97.2		—81.0		—75.9	
P_1	0.	—14.2	—13.9	— 2.0	—17.5	0.	—13.9	— 2.0	—16.7	+ **0.8**	—16.9	— 5.0	—**19.7**	— 2.2
P_2	—30.7	—47.2	—**78.2**	—27.9	—43.7	—48.1	—**78.2**	—27.9	—44.2	— 48.6	—74.1	—23.9	— 40.2	—44.6
T_1	+ 7.3	— 4.9	— 4.5	+ 4.3	— 8.3	+ 7.3	— 4.5	+ 4.3	— 7.9	+ **7.7**	— 8.8	0.	—**12.3**	+ 3.4
T_2	+14.1	+13.4	+ 13.7	+11.6	+ 9.5	+14.1	+ 13.7	+11.6	+10.4	+**15.0**	+ 9.8	+ 8.0	+ 6.7	+11.5
T_3	+19.6	+25.7	+**26.1**	+17.2	+23.0	+19.6	+**26.1**	+17.2	+23.3	+ 19.9	+22.6	+13.7	+ 19.8	+16.4
T_4	+25.0	+40.1	+**40.4**	+23.0	+36.9	+25.1	+**40.4**	+23.0	+37.6	+ 25.8	+38.7	+23.5	+ 34.7	+22.8
T_5	+29.6	+50.1	+**50.4**	+27.6	+47.7	+29.6	+**50.4**	+27.6	+48.1	+ 29.9	+47.4	+26.2	+ 45.1	+27.0
T_6	0.	0.	0.		+**21.9**		0.		+**21.9**		0.		+**21.9**	
T_7	0.	0.	+**38.6**		0.		+**38.6**		0.		+**38.6**		0.	
S_1	0.	+18.3	+ 17.9	+ 2.6	+22.5	0.	+ 17.9	+ 2.6	+21.6	— **1.0**	+21.7	+ 6.5	+**25.4**	+ 2.6
S_2	— 6.3	+ 4.2	+ 3.9	— 3.7	+ 7.2	— 6.3	+ 3.9	— 3.7	+ 6 8	— **6.6**	+ 7.7	0.	+**10.6**	— 2.7
S_3	—12.3	—11.7	— 12.0	—10.1	— 8.2	—12.3	— 12.0	—10.1	— 9.0	—**13.0**	— 8.8	— 6.9	— 6.0	— 9.9
S_4	—17.2	—22.6	—**22.8**	—15.1	—20.1	—17.2	—**22.8**	—15.1	—20.4	— 17.5	—19.8	—12.0	— 17.4	—14.8
S_5	—22.2	—35.5	—**35.7**	—20.4	—32.6	—22.2	—**35.7**	—20.4	—33.3	— 22.8	—32.0	—17.7	— 30.7	—20.3

The figures in heavy type show the maximum strains in the several members.

TABLE (*B*) OF STRAINS WITH THE SNOW LOAD REMOVED.

Pieces.	*Case I.*	*Case II.*	*Case III.* (*a*)		*Case III.* (*b*)		*Case IV.* (*a*)		*Case IV.* (*b*)		*Case V.* (*a*)		*Case V.* (*b*)	
			Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*	Span *AB.*	Span *CD.*
U_1	0.	—11.2	—11.0	— 0.3	—15.8	0.	—11.0	— 0.3	—13.5	+ **2.3**	—13.4	— 2.9	—17.8	— 2.0
U_2	+ 7.6	—15.6	—14.9	+ **6.8**	**—27.7**	+ 7.6	—14.9	+ **6.8**	—21.7	**+13.6**	—22.7	— 1.2	**—34.5**	+ 0.6
U_3	+21.7	+ **0.3**	+ **1.4**	**+20.6**	—19.1	+21.7	+ **1.4**	**+20.6**	— 9.4	**+31.4**	—10.9	+ 8.2	**—29.1**	+11.5
U_4	+40.7	**+28.8**	**+30.3**	+39.2	+ 3.7	+40.7	**+30.3**	+39.2	+16.3	**+53.2**	**+13.9**	**+22.0**	— **9.9**	+27.0
U_5	+64.1	+71.1	+72.9	+62.2	+40.3	+64.1	+72.9	+62.2	+55.7	+79.4	+53.2	+41.7	+24.2	+47.9
U_6	+77.9	+97.5	+75.9		+64.6		+75.9		+81.0		+54.6		+47.0	
L_1	— 4.1	+ 2.4	+ 2.1	— **3.8**	+ 6.4	— 4.1	+ 2.1	— **3.8**	+ 4.4	— **6.1**	+ 5.0	— **0.9**	+ 9.0	— 1.5
L_2	—11.6	+ 5.1	+ 4.3	**—10.9**	+17.5	—11.4	+ 4.3	**—10.9**	+11.3	**—17.8**	+12.2	— **3.0**	**+23.8**	— 5.3
L_3	—25.1	— **8.1**	— **9.2**	**—24.0**	+10.9	—25.1	— **9.2**	**—24.0**	+ 1.4	**—34.6**	+ 3.3	**—11.5**	**+21.4**	—14.5
L_4	—44.1	**—37.8**	**—39.3**	**—42.6**	—12.4	—44.1	**—39.3**	**—42.6**	**—25.1**	**—56.7**	**—23.8**	**—26.4**	+ 0.8	—30.6
L_5	—67.0	—77.9	—79.7	—65.3	—47.4	—67.0	—79.7	—65.3	—62.8	—82.3	—60.7	—45.6	—31.0	—50.5
L_6	—77.9	—97.5	—99.5		—77.9		—99.5		—94.4		—78.2		—60.3	
P_1	0.	—12.4	—12.1	— 0.3	—17.5	0.	—12.1	— 0.3	—14.9	+ **2.5**	—15.1	— 3.2	—19.7	— 2.2
P_2	—25.7	—44.5	—75.5	—25.3	—38.5	—42.9	—75.5	—25.3	—41.5	—45.9	—71.4	—21.2	—35.1	—39.5
T_1	+ 6.1	— 3.5	— 3.1	+ **5.7**	— **9.5**	+ 6.1	— 3.1	+ **5.7**	— 6.5	+ **9.1**	— 7.4	+ **1.4**	**—13.5**	+ 2.2
T_2	+11.7	+13.2	+13.5	+11.4	+ 7.1	+11.7	+13.5	+11.4	+10.2	+14.8	+ 9.6	+ 7.8	+ 4.3	+ 9.1
T_3	+16.3	+24.6	+24.9	+16.0	+19.7	+16.3	+24.9	+16.0	+22.1	+18.8	+21.4	+12.5	+16.5	+13.1
T_4	+20.9	+37.7	+38.0	+20.6	+32.7	+20.9	+38.0	+20.6	+35.2	+23.4	+36.3	+21.1	+30.5	+18.6
T_5	+21.7	+46.9	+47.2	+24.4	+42.8	+24.8	+47.2	+24.4	+44.9	+26.8	+44.3	+23.0	+40.6	+22.5
T_6	0.	0.	0.		+21.9		0.		+21.9		0.		+21.9	
T_7	0.	0.	+38.6		0.		+38.6		0.		+38.6		0.	
S_1	0.	+16.0	+15.7	+ 0.4	+22.5	0.	+15.7	+ 0.4	+19.3	— **3.3**	+19.5	+ 4.2	+25.4	+ 2.6
S_2	— 5.2	+ 3.1	+ 2.7	— 4.9	+ 8.2	— 5.2	+ 2.7	— **4.9**	+ 5.6	— **7.8**	+ 6.5	— **1.2**	**+11.6**	— 1.7
S_3	—10.2	—11.5	—11.8	— 9.9	— 6.2	—10.2	—11.8	— 9.9	— 8.8	—12.9	— 8.7	— 6.8	— 4.0	— 7.9
S_4	—14.3	—21.5	—21.8	—14.1	—17.3	—14.3	—21.8	—14.1	—19.3	—16.5	—18.7	—11.0	—14.5	—11.9
S_5	—18.5	—33.4	—33.6	—18.2	—28.9	—18.5	—33.6	—18.2	—31.2	—20.7	—29.8	—15.6	—27.0	—16.6

☞ The figures in heavy type show the maximum strains in the several members, as compared with the strains of the previous table.

APPENDIX.

LIST OF BOOKS AND ARTICLES IN TECHNICAL JOURNALS, RELATING TO THE SUBJECT OF CONTINUOUS GIRDERS (IN PART OR IN WHOLE), IN THE GERMAN, FRENCH, AND ENGLISH LANGUAGES, 1854–1874.

[Without wishing to presume the part of an adviser, the writer yet offers the opinion that the starred works are especially to be recommended to those desirous of further pursuing the subject of Continuous Girders. The annexed list is doubtless incomplete.]

***Weyrauch*, *Dr. Phil. Jacob I.*, Ingenieur, &c. Allgemeine Theorie und Berechnung der Continuirlichen und Einfachen Träger. Leipzig: B. G. Teubner. 1873.

**Ritter*, *August*, Prof. &c. Elementare Theorie und Berechnung eiserner Dach- & Brücken-Constructionen. (Contains, amongst other matters, a peculiar *kind* of continuous girder.) Hannover: Carl Rümpler. 1873.

**Laissle & Schuebler*. Der Bau der Brücken-Träger. Stuttgart: Paul Neff. 2 vols. 1869 & 1870. (This work has also appeared in a French translation.)

Winkler, *Dr. E.* Elasticität und Festigkeit. Prag: H. Dominicus. 1867.

ZEITSCHRIFT DES ARCHT. & ING. VEREIN ZU HANNOVER.

**Mohr* (now Prof. &c. in Stuttgart)	1860, 1862, & 1868.
Köpke, *C.*, Ingr. &c.	1856 & 1857.

ZEITSCHRIFT FÜR BAUWESEN, BERLIN.

Quensel, *O.*	1865.
Schwedler, *J. W.*	1862.
Winkler, *E. O.*	1860.

ZEITSCHRIFT DES OESTR. ARCHT. & ING. VEREIN.

Winkler, *Prof.* &c.	1872.

DER CIVIL INGENIEUR.

Fränkel, *W.*	1868, p. 271.
—— ——	1860, Heft 3 & 4.
Hallbauer	1858, ,, 5 & 6.
—— ——	1858, ,, 2.

Zeitschrift des Vereins Deutscher Ingenieure.

Grashof . . . . 1857.

Zeitschrift des Bayerischen Archt. & Ing. Verein.

Gerber. (A Patent Continuous Girder.) . . . . 1870.

Allgemeine Bauzeitung.

Schmidt, *Heinrich* . . . . 1868.

De Montdesir. (Strains in Continuous Trusses of Iron Bridges.) 2d Ed. . . . . 1873.

*_Bresse._ Cours de Mécanique Appliquée. 1re & 3me partie. Paris. 1859 & 1865.

Annales des Ponts et Chausées.

*_Renaudot_ . . . . 1866, p. 311.

Pierre . . . . 1871, p. 44.

Albaret . . . . 1866, p. 53.

"Comptes Rendus."

Clapeyron . . . . Dec. 1857.

Molinos & Pronnier. Construction des Ponts Métalliques. Paris. 1857.

Annales des Travaux Publics de Belgique.

De Clerq . . . . 1855–56.

[*Navier.* Résumé des Leçons données à l'École des P. & C. &c. 2d Ed. Paris. 1833.]

Minutes of the Proceedings of the Inst. Civ. Engrs.

Bell, *W.* . . . . Vol. 32, p. 171.

*_Stoney_, _E. W._ . . . . ,, 29, p. 382.

*_Heppel_, _J. M._ . . . . ,, 19, p. 625.

*_Barton_, _James_ . . . . ,, 14, p. 443.

Contains a description of the celebrated, and, considering the time it was built, *wonderfully perfect*, Boyne Bridge at Drogheda, Ireland, 3 continuous spans of 141, 267, and 141 feet, opened for traffic, double track R.R., in April, 1855.

Pole, *W.* . . . . Vol. 9, p. 261.

Hemans, *G. W.* . . . . ,, 3, p. 65.

The Philosophical Magazine, London.

*_Heppel_, _J. M._ . . . . Vol. 40, p. 446.

Stoney, *B. B.* Theory of Strains in Girders, &c. London: Longmans, Green, & Co. . . . . 1873.

Rankine, *W. J. M.* Civil Engineering . . . . pp. 287–292.

Humber, *William.* Strains in Girders. Am. Ed. New York: Van Nostrand . . . . 1870.

Clark, *Edwin.* Brittania and Conway Tubular Bridges. London. 1850.

Moseley, *Henry.* Mechanical Principles of Engineering and Architecture (1840–42). Am. Ed. By D. H. Mahan, Professor at West Point. New York: J. Wiley & Son. 1869. (Continuous beams are treated of in this work by the name of "Breast summers.")

Chanute, *O.*, and *G. Morison.* Kansas City Bridge. pp. 86, 87, 108–114.

JOURNAL OF THE FRANKLIN INSTITUTE.

Frizell, *J. P.* . . . . 3d Series, Vol. 64, 1872.

TRANSACTIONS ENGINEERS' CLUB OF ST. LOUIS.

Smith, Charles A. Girders Continuous over Supports . . Dec. 4, 1872.

VAN NOSTRAND'S ECLECTIC ENGINEERING MAGAZINE.

Eddy, *C. H.* . . . . p. 552, 1874.

DUTCH LANGUAGE.

Tijdschrift van het Koninklijk Instituut van Ingenieurs.* 's Gravenhage . . . . 1870 & 1871.

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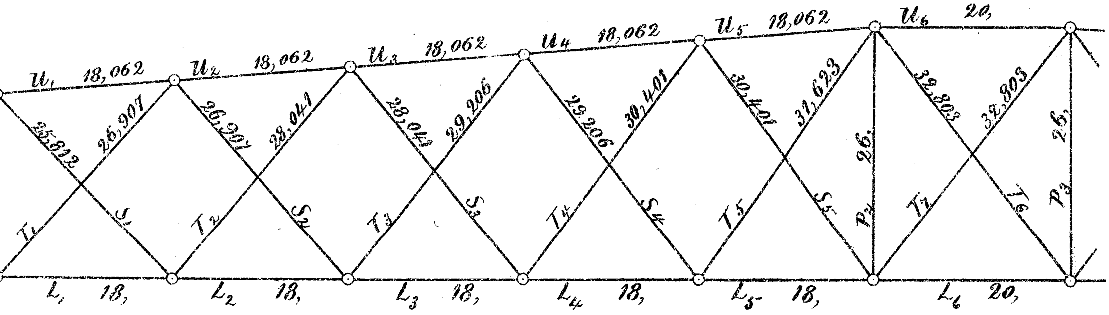
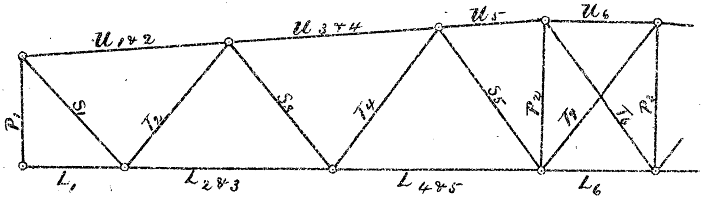
* The writer does not know the exact nature of these articles.

PLATE 1.

Names and lengths of the Members.



First System.



Second System.

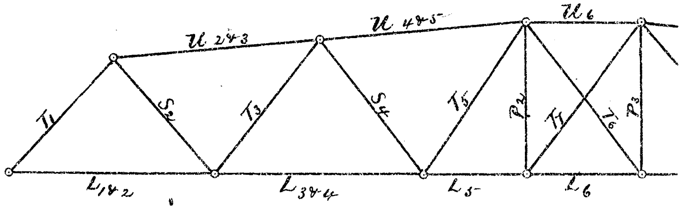


PLATE II.

Calculation of the lever-arms.

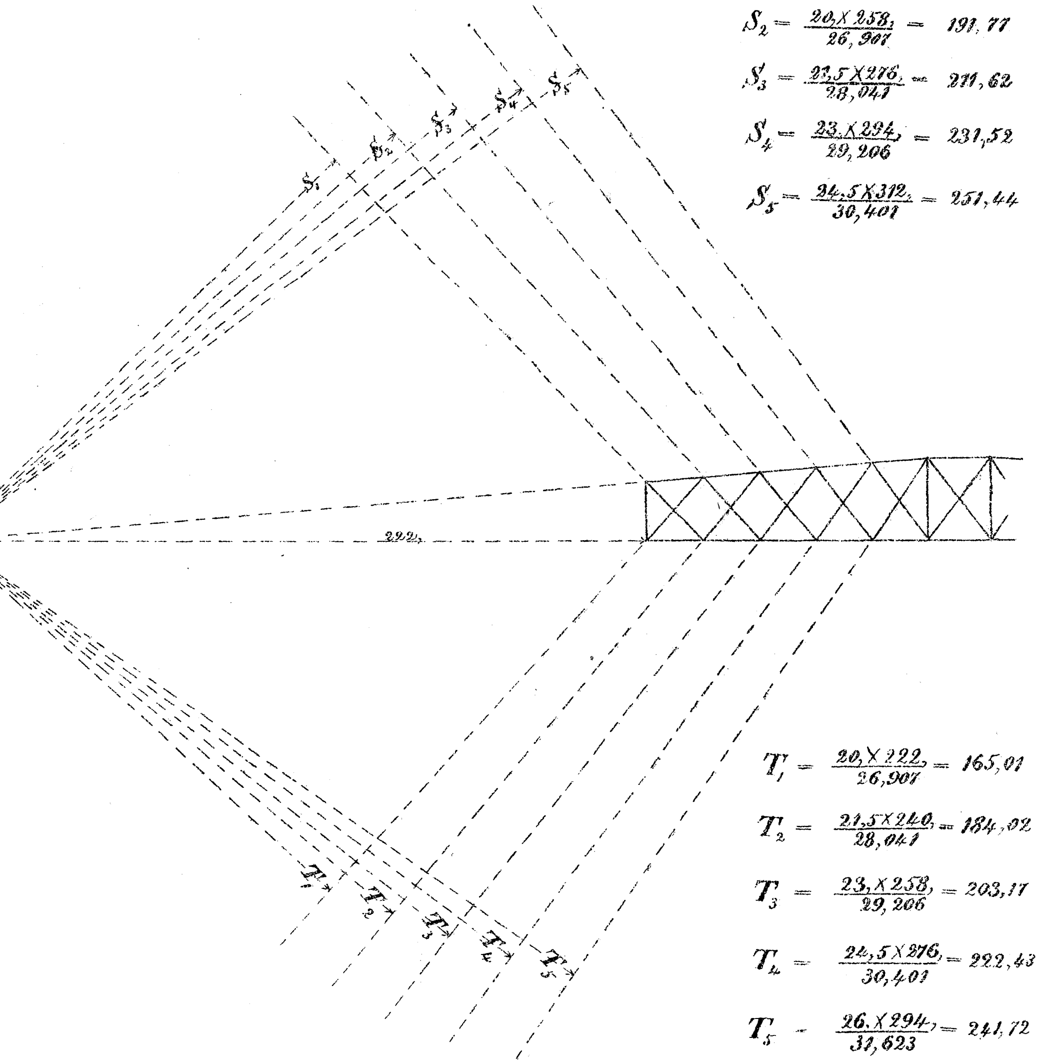
$$S_1 = \frac{18{,}5 \times 240,}{25{,}812} = 172{,}005$$

$$S_2 = \frac{20, \times 258,}{26{,}907} = 191{,}77$$

$$S_3 = \frac{21{,}5 \times 276,}{28{,}041} = 211{,}62$$

$$S_4 = \frac{23, \times 294,}{29{,}206} = 231{,}52$$

$$S_5 = \frac{24{,}5 \times 312,}{30{,}401} = 251{,}44$$



$$T_1 = \frac{20, \times 222,}{26{,}907} = 165{,}01$$

$$T_2 = \frac{21{,}5 \times 240,}{28{,}041} = 184{,}02$$

$$T_3 = \frac{23, \times 258,}{29{,}206} = 203{,}17$$

$$T_4 = \frac{24{,}5 \times 276,}{30{,}401} = 222{,}43$$

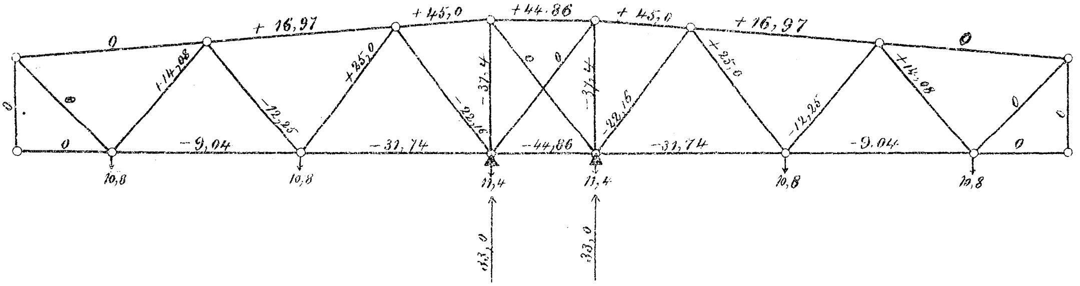
$$T_5 = \frac{26, \times 294,}{31{,}623} = 241{,}72$$

PLATE III.

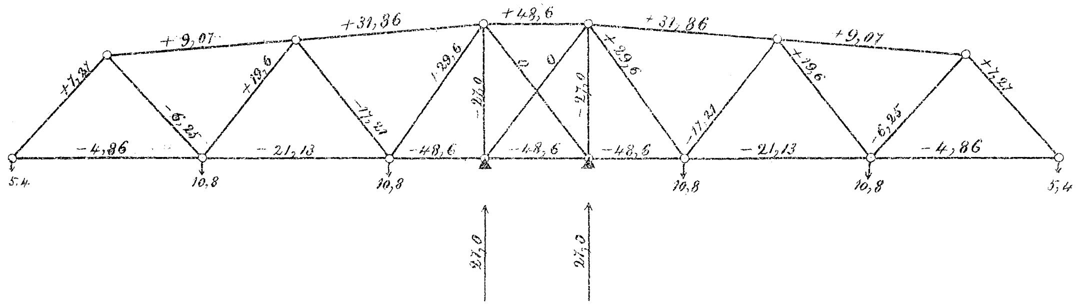
Case I.

Bridge open: loaded with dead + snow.

Strains in the first System.



Strains in the second System.



Total Strains.

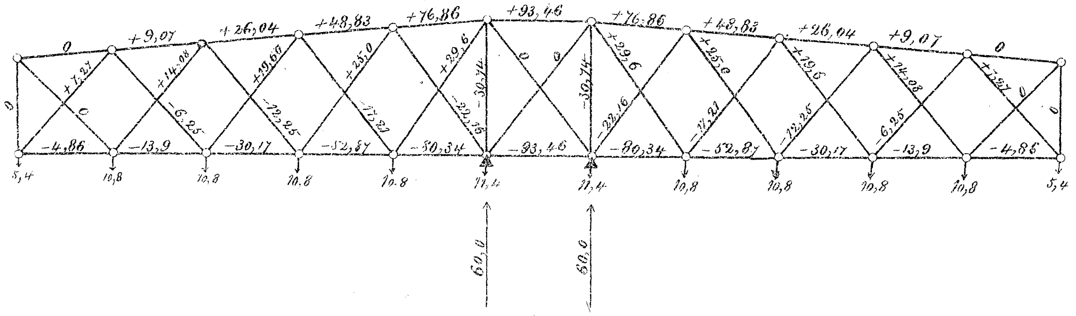
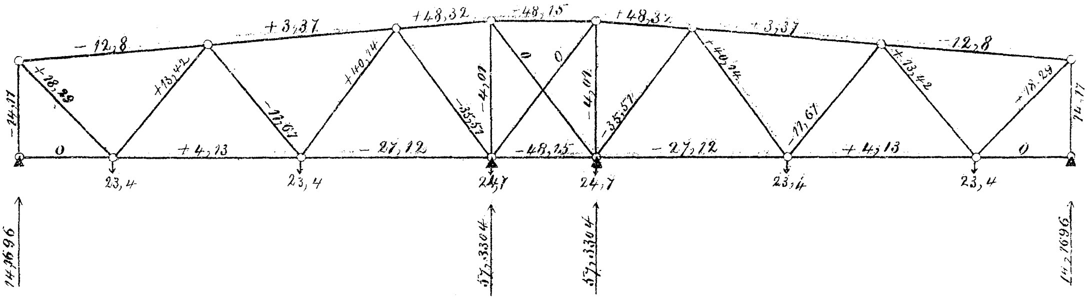


PLATE IV

Case II.

Bridge closed: loaded with dead + snow + live; reactions at the ends due to snow + live only.

Strains in the first System



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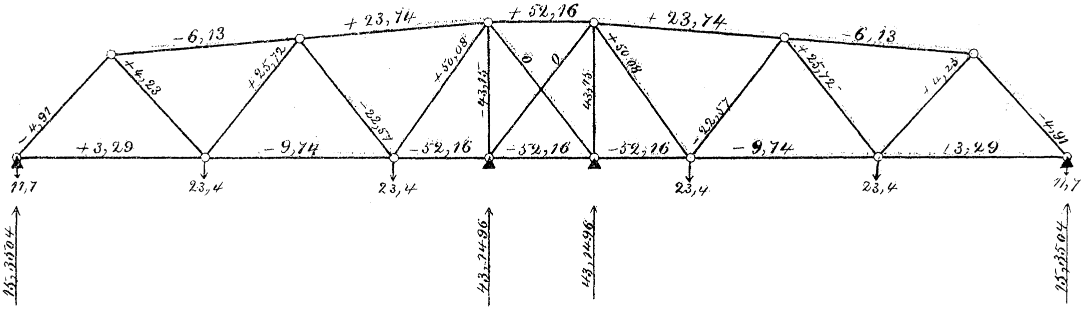


Total Strains.

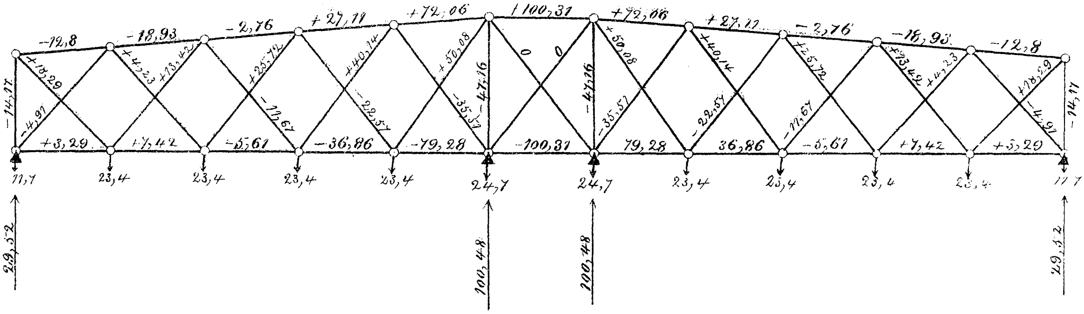
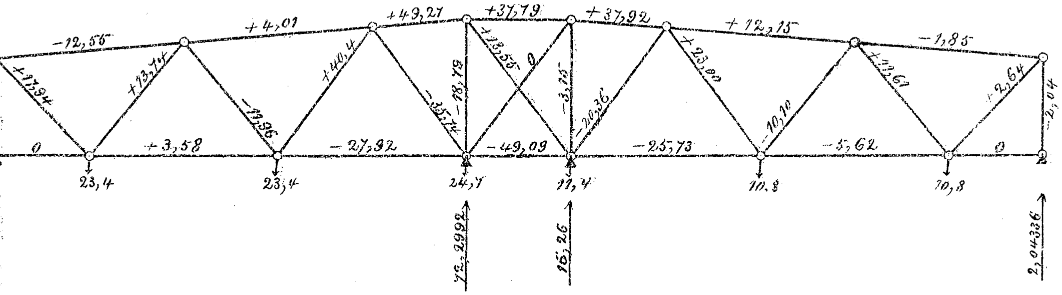


PLATE V.

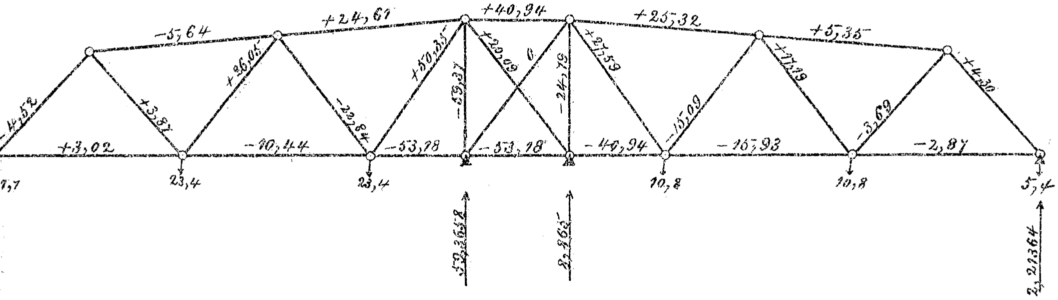
Case III(a).

[Also Case IV(a) in this example, because D is plus.] Centre supports fixed; ends free; Am loaded with dead + snow + live; mD with dead + snow; reactions at ends due to snow + live only.

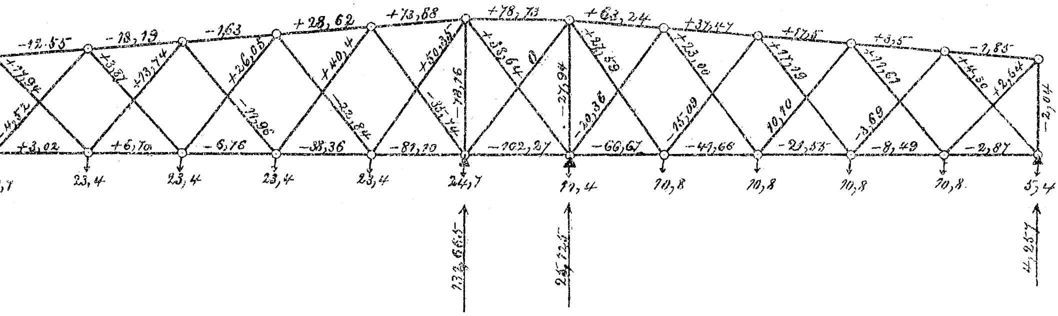
Strains in the first System.



Strains in the second System.



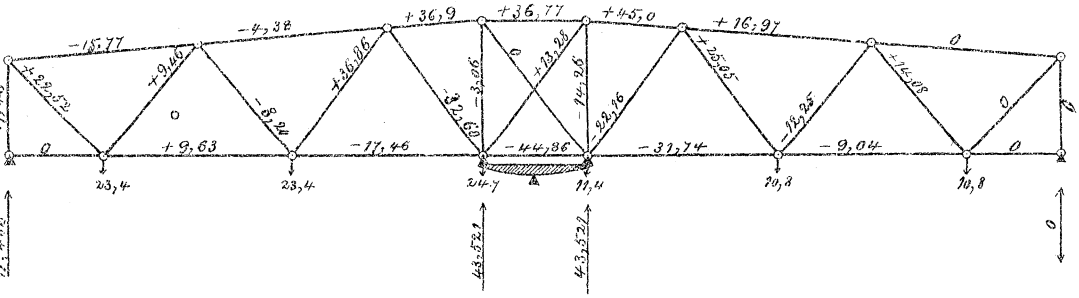
Total Strains.



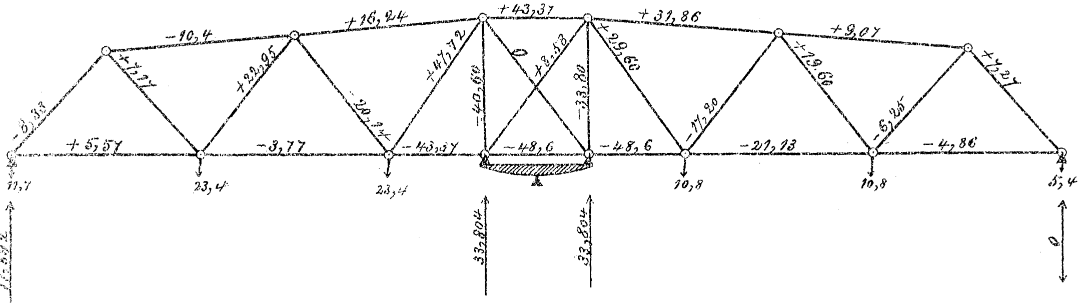
Case III(b).

Centre supports pivotted; ends free. Am loaded with dead + snow + live; mD with dead + snow; reactions at ends due to snow + live only.

Strains in the first System.



Strains in the second System.



Total Strains.

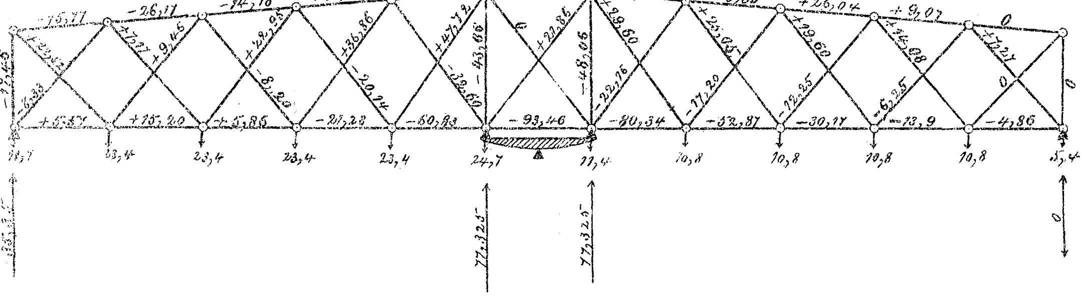
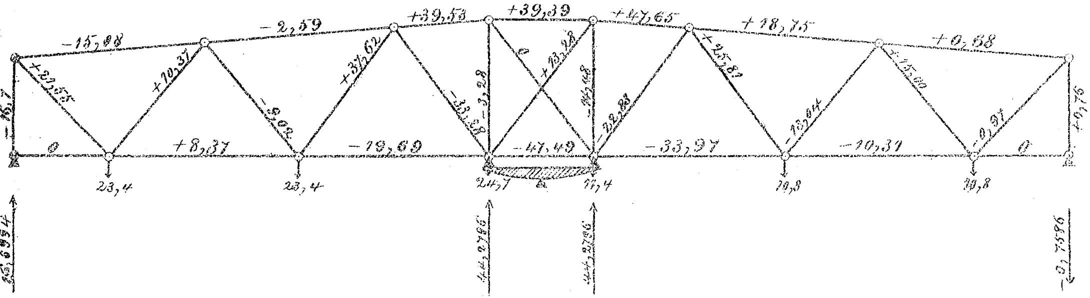


PLATE VII.

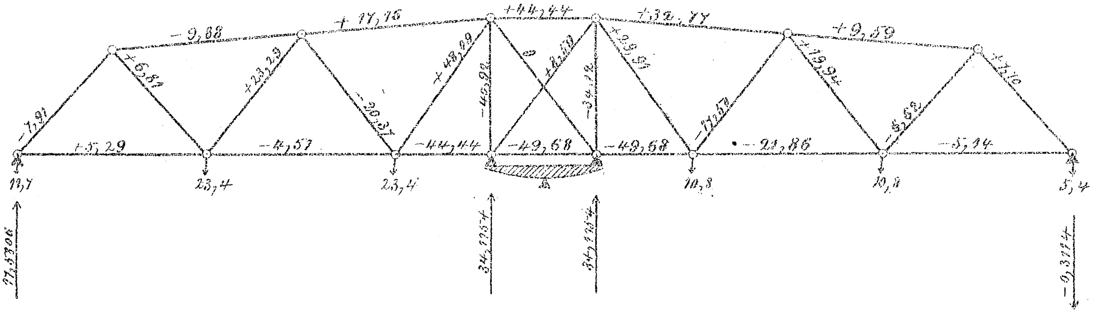
Case IV(b).

Centre supports pivotted; ends latched; Am loaded with dead + snow + live; mD with dead + snow; reactions at ends due to snow + live only.

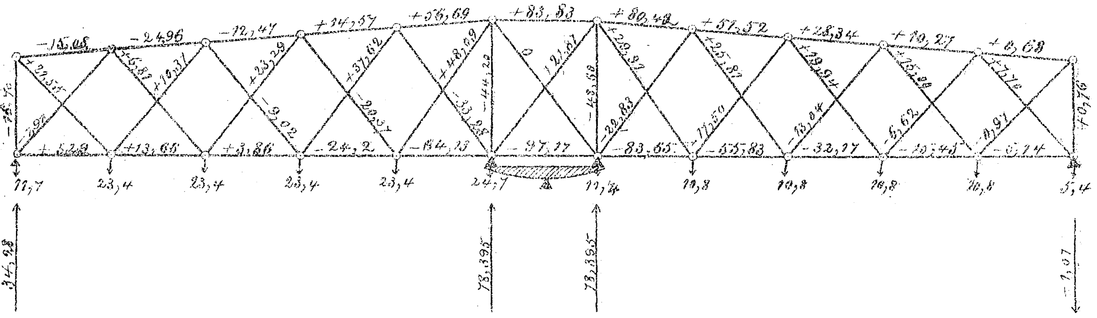
Strains in the first System.



Strains in the second System.



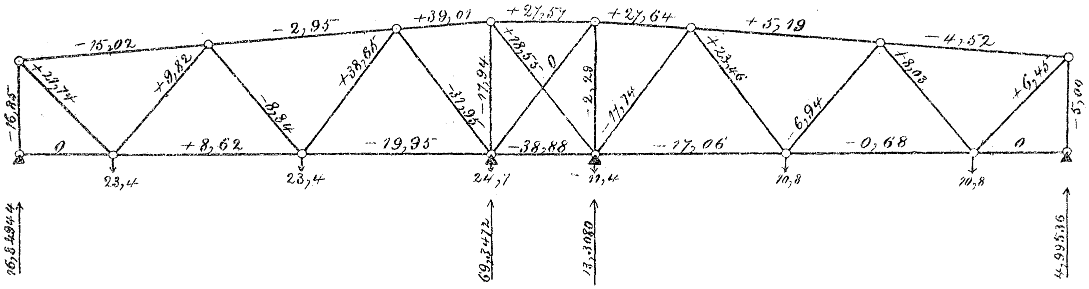
Total Strains.



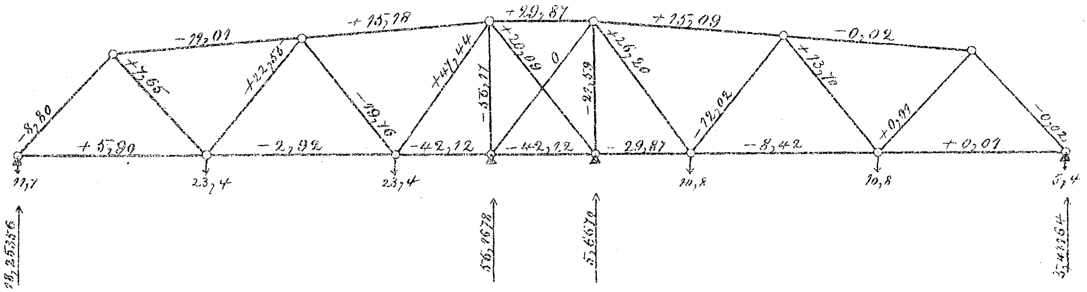
Case V(a).

Centre supports fixed; ends lifted; Am loaded with dead + snow + live; mD with dead + snow; reactions at ends due to live + snow + $\frac{2}{3}$ dead.

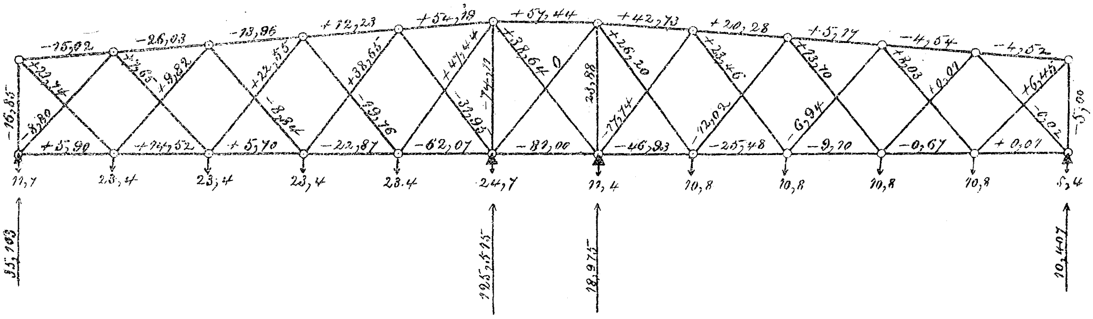
Strains in the first System.



Strains in the second System.



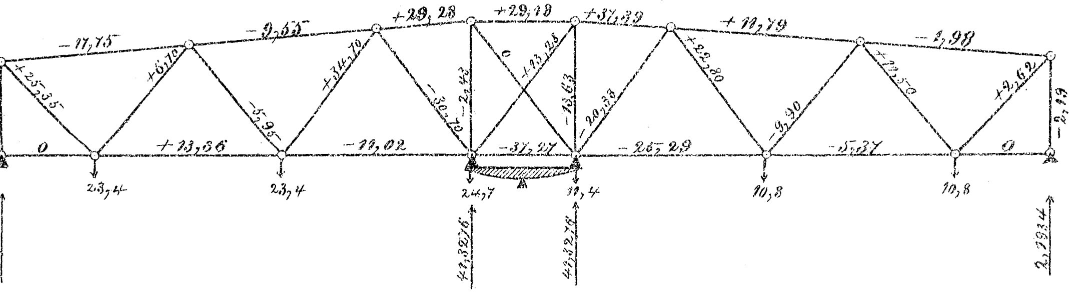
Total Strains.



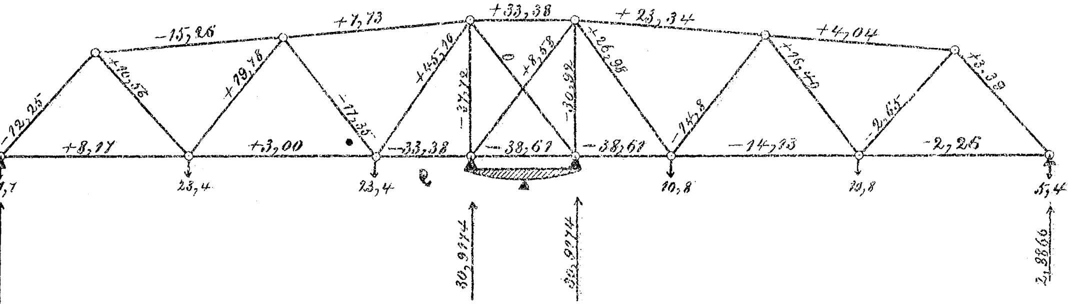
Case V(b).

Centre supports pivotted; ends lifted; Am loaded with dead + snow + live; mD loaded with dead + snow; reactions at ends due to snow + live + $\frac{2}{3}$ dead.

Strains in the first System.



Strains in the second System.



Total Strains.